Dawodu Abdulsalam Oluwafemi
Computer Science
19/SCI01/039

1. $x^2 \sin x \, \partial x$

Solution

$\int x^2 \sin x \, dx$

$u = x^2$ $\quad$ $dv = \sin x$

$du = 2x \, dx$ $\quad$ $v = -\cos x$

$\int u \, dv = uv - \int v \, du$

$= x^2(-\cos x) - \int(-\cos x)(2x) \, dx$

$= -x^2 \cos x - \int -2x \cos x \, dx$

$\int -2x \cos x \, dx$

$u = -2x$ $\quad$ $dv = \cos x$

$du = -2 \, dx$ $\quad$ $v = \sin x$

$\int u \, dv = uv - \int v \, du$

$= (-2x)(\sin x) - \int(\sin x)(-2) \, dx$

$= -2x \sin x - \int -2 \sin x \, dx$

$= -2x \sin x - (-2) \int \sin x \, dx$

$= -2x \sin x + 2 \int \sin x \, dx$

$= -2x \sin x + 2(-\cos x)$

$= -2x \sin x - 2 \cos x$

$\therefore \int x^2 \sin x \, dx = -x^2 \cos x - [-2x \sin x - 2 \cos x]$

$= -x^2 \cos x + 2x \sin x + 2 \cos x + c$

2) $3te^{2t} \, dt$

Solution

$\int 3te^{2t} \, dt$

$u = 3t$ $\quad$ $dv = e^{2t}$

$du = 3 \, dt$ $\quad$ $v = \frac{1}{2} e^{2t}$

$\int u \, dv = uv - \int v \, du$

$= (3t)\left(\frac{1}{2} e^{2t}\right) - \int \frac{1}{2} e^{2t} \times 3 \, dt$

$= 3t \times \frac{e^{2t}}{2} - \frac{3}{2} \int e^{2t} \, dt$

$= \frac{3te^{2t}}{2} - \frac{3}{2}\left[\frac{1}{2}e^{2t}\right]$

$= \frac{3te^{2t}}{2} - \frac{3}{4}e^{2t}$

$= \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + c$

3) $2x^2 \ln x \, dx$

Solution

$\int 2x^2 \ln x \, dx$

$2\int x^2 \ln x \, dx$

$u = \ln x \quad dv = x^2 \quad du = 1/x \quad v = x^3/3$

$\int u\,dv = uv - \int v\,du$

$= 2\left[\ln(x) \times x^3/3 - \int \frac{x^3}{3} \times 1/x \, dx\right]$

$= 2\left[\ln(x) \times x^3/3 - \int \frac{x^2}{3} \, dx\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2 \, dx\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3} \times \frac{x^3}{3}\right] + c$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{x^3}{9}\right] + c$

$= \frac{2x^3 \ln(x)}{3} - \frac{2x^3}{9} + c$

$= \left[\frac{2x^3}{3}\left(\ln(x) - \frac{1}{3}\right)\right] + c$

4) $(2x - 3x^2)/(1-x)\,dx$

$\int \frac{2x - 3x^2}{1-x}\,dx$

divide the fraction into 2 fractions

$\int \frac{2x}{1-x} - \frac{3x^2}{1-x}\,dx$

$$= \int \frac{2x}{1-x}\,dx - \int \frac{3x^2}{1-x}\,dx$$

The first fraction

$$\int \frac{2x}{1-x}\,dx$$

$u = 1-x$

$du/dx = -1$

$\frac{du}{-1} = \frac{+dx}{\cancel{+}1}$

$dx = -du$

$$\int \frac{2x}{1-x}\,dx = \int \frac{2x}{u}\,dx = \int \frac{2x}{u} \cdot -du = -2\int \frac{x}{u}\,du$$

Recall, $u = 1-x$

$x = 1-u$

$$= -2\int \frac{x}{u}\,du = -2\int \frac{(1-u)}{u}\,du$$

$$= -2\left[\int \frac{1}{u} - \frac{u}{u}\,du\right] = -2\left[\int \frac{1}{u}\,du - \int \frac{u}{u}\,du\right]$$

$$= -2\left[\ln(u) - \int 1\,du\right]$$

$$= -2\left[\ln(u) - u\right]$$

$$= -2\left[\ln(1-x) - (1-x)\right]$$

$$= -2\left[\ln(1-x) - 1 + x\right]$$

$$\therefore \int \frac{2x}{1-x}\,dx = -2\ln(1-x) + 2 - 2x$$

The second fraction

$$\int \frac{3x^2}{1-x}\,dx$$

$$3\int \frac{x^2}{1-x}\,dx$$

$u = 1-x$

$du/dx = -1$ $\quad du = -dx \quad dx = -du$

$$\int \frac{3x^2}{1-x}\,dx = 3\int \frac{x^2}{u} \cdot -du = -3\int \frac{x^2}{u}\,du$$

Recall, $u = 1-x$

$x = 1-u$

$$= -3\int \frac{x^2}{u}\,du = -3\int \frac{(1-u)^2}{u}\,du = -3\int \frac{1-2u+u^2}{u}\,du$$

$$= -3\left[\int \frac{1}{u} - \frac{2u}{u} + \frac{u^2}{u}\,du\right] = -3\left[\int \frac{1}{u}\,du - \int \frac{2u}{u}\,du + \int \frac{u^2}{u}\,du\right]$$

$$= -3\left[\ln(u) - \int 2\,du + \int u\,du\right]$$

$$= -3\left[\ln(u) - 2u + \frac{u^2}{2}\right]$$

$$= -3\ln(u) + 6u - \frac{3u^2}{2}$$

$$= -3\ln(1-x) + 6(1-x) - \frac{3(1-x)^2}{2}$$

$$= -3\ln(1-x) + 6(1-x)/_1 - \frac{3(1-2x+x^2)}{2}$$

$$= -3\ln(1-x) + \frac{(6\times 2)(1-x) - 3(1-2x+x^2)}{2}$$

$$= -3\ln(1-x) + \frac{12(1-x) - (3-6x+3x^2)}{2}$$

$$= -3\ln(1-x) + \frac{12-12x-3+6x-3x^2}{2}$$

$$\int 3x^2/_{1-x}\,dx = -3\ln(1-x) + \frac{9-6x-3x^2}{2}$$

So,

$$\int 2x/_{1-x}\,dx - \int 3x^2/_{1-x}\,dx = -2\ln(1-x) + 2 - 2x - \left[-3\ln(1-x) + \frac{9-6x-3x^2}{2}\right]$$

$$= -2\ln(1-x) + 2 - 2x + 3\ln(1-x) - \frac{9+6x+3x^2}{2}$$

$$= \frac{2-2x}{1} - \frac{9+6x+3x^2}{2} - 2\ln(1-x) + 3\ln(1-x)$$

$$= \frac{2(2-2x) - 9 + 6x + 3x^2}{2} + \ln(1-x) + c$$

$$= \frac{4-4x-9+6x+3x^2}{2} + \ln(1-x) - c$$

$$\int \frac{2x-3x^2}{(1-x)}\,dx = \frac{3x^2+2x-5}{2} + \ln(1-x) + c$$