

$$1.) 3te^{2t}$$

Solution

$$\text{let } u = 3t$$

$$\text{and } dv = e^{2t}$$

$$\frac{du}{dt} = 3$$

$$\times$$

$$du = 3dt$$

$$\int dv = \int 3e^{2t}$$

$$v = \frac{e^{2t}}{2}$$

$$\text{Using } uv - \int v du = \int u dv$$

$$= 3t \left(\frac{e^{2t}}{2} \right) - \int \frac{e^{2t}}{2} \times 3dt$$

$$3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \int 3e^{2t} dt$$

$$3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \times \frac{3e^{2t}}{2} + C$$

$$\left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} \right] + C$$

$$2) \int x^2 \sin x$$

$$\text{let } u = x^2 \text{ and } dv = \sin x$$

$$\frac{du}{dx} = 2x \text{ and } v = -\cos x$$

$$\text{using } uv - \int v du$$

$$(x^2)(-\cos x) - \int (-\cos x)(2x dx)$$

$$-x^2 \cos x - \int -2x \cos x dx$$

$$\left[\begin{array}{l} \text{let } u = -2x \text{ and } du = \cos x \\ \frac{du}{dx} = -2 \text{ and } v = \sin x \end{array} \right]$$

$$\therefore (-2x)(\sin x) - \int (\sin x)(-2) dx$$

$$-2x \sin x - (-2) \int \sin x dx$$

$$-2x \sin x - (-2) \left(-\cos x \right) + C$$

$$-2x \sin x - 2 \cos x + C$$

$$\therefore \int x^2 \sin x = -x^2 \cos x - 2x \sin x - 2 \cos x + C$$

Using $UV - \int U du = \int v du$

$$= 3t \left(\frac{e^{2t}}{2} \right) - \int \frac{e^{2t}}{2} \times 3 dt$$

$$= 3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \int 3e^{2t} dt$$

$$= 3t \left(\frac{e^{2t}}{2} \right) - \frac{1}{2} \times \frac{3e^{2t}}{2} + C$$

$$\left[\frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} \right] + C$$

③ $2x^2 \ln x \, dx$

let $u = 2x^2$, $du = 4x \, dx$

$\frac{du}{dx} = 4x$ $v = \frac{1}{x}$

$du = 4x \, dx$ $u = \frac{1}{x}$

Using $UV - \int U du = \int v du$

$$= 2x^2 \left(\frac{1}{x} \right) - \int \frac{1}{x} \times 4x \, dx$$

$$= \frac{2x^2}{x} - \int \frac{4x}{x} \, dx$$

$$= 2x - 4 + C$$

$$4.) \frac{2x - 3x^2}{1-x}$$

$$\begin{array}{r} 2x - x^2 \\ 1-x \overline{) 2x - 3x^2} \\ \underline{-2x - 2x^2} \\ -x^2 \\ \underline{-x^2 + x^3} \\ -x^3 \end{array}$$

which can now be

$$\int (2x - x^2) dx + \int \frac{x^3}{1-x} dx$$

$$= \frac{2x^2}{2} - \frac{x^3}{3} + x^3 \ln(1-x) \quad \text{||}$$