MEE 322
Assignment 3

Solution

Given: $\mu = 0.9 Ns/m^2$
$\rho = 1260 kg/m^3$
$L = 65 m$
$D = 10 mm = 0.01 m$
$q = 180 Lit/min = 0.003 m^3/s$

a) from continuity equation,

$$q = A \cdot u$$

where $A = \frac{\pi D^2}{4} = \frac{\pi \times (0.01)^2}{4} = 7.855 \times 10^{-5} m^2$

$\therefore u = \frac{q}{A} = \frac{0.003}{7.855 \times 10^{-5}} = 38.19 m/s$

$\therefore Re = \frac{\rho u D}{\mu} = \frac{1260 \times 38.19 \times 0.01}{0.9} = 534.66$

$\therefore$ Because $Re < 2000$, the flow is laminar

b) $\Delta P = \frac{32 \mu u L}{D^2} = \frac{32 \times 0.9 \times 38.19 \times 65}{(0.01)^2} = \frac{71491.68}{0.0001}$

$= 7.15 \times 10^8 N/m^2$

2. Given : $\mu = 800\,cp = 0.8\,Ns/m^2$

$G = 0.85$

$\therefore \rho = 850\,kg/m^3$

$dp = 2000 \times 10^3\,N/m^2$

$D = 65\,mm = 0.065\,m$

$L = 95\,m$

(a) $A = \frac{\pi D^2}{4} = \frac{\pi \times (0.065)^2}{4} = 3.319 \times 10^{-3}\,m^2$

$\frac{dp}{dx} = \frac{-2000 \times 10^3}{95} = -21.05 \times 10^3$

Rate of flow $= Q = A \times \bar{u}$

where $\bar{u} = -\frac{1}{8\mu}\frac{\partial p}{\partial x}R^2$

$= \left(\frac{-1}{8 \times 0.8}\right)\left(-21.05 \times 10^3\right)(0.0325)^2$

$\bar{u} = 3.474\,m/s$

$\therefore Q = 3.474 \times 3.319 \times 10^{-3} = 0.0115\,m^3/s.$

(b) Centre line velocity $= u_{max}$

and $u_{max} = 2 \times \bar{u}$

$= 2 \times 3.474$

$= 6.948\,m/s.$

(c) Total frictional drag, F_D

$F_D = \tau_o \pi D L$

where $\tau_o = -\frac{\partial p}{\partial x} \cdot \frac{r}{2}$

$= 21.05 \times 10^3 \times \frac{0.0325}{2}$

$\tau_o = 342.0625 N/m^2$

$\therefore F_D = 342.0625 \times \pi \times 0.065 \times 95$

$F_D = 6636.645 N \simeq 6.637 kN.$

d) Power required to maintain flow.

$P = F_D \times \bar{u}$

$= 6636.645 \times 3.474$

$\therefore P = 23055.7$ Watts

$P = 23 kW.$

e) Velocity gradient at the pipe wall

$\tau_o = \mu \frac{\partial u}{\partial y}$ @ $y = 0$

$\frac{\partial u}{\partial y} = \frac{\tau_o}{\mu} = \frac{342.0625}{0.8} = 427.58 \, s^{-1}.$

f) Velocity and shear stresss 60mm from wall.

$$u = -\frac{1}{4\mu} \cdot \frac{\partial p}{\partial x} \cdot (R^2 - r^2)$$

but $y = R - r$ and $y = 60mm = 0.06m$

$$\therefore 0.06 = 0.065 - r$$
$$r = 0.005$$

$$\therefore u = \frac{-1}{(4 \times 0.8)} \left(-21.05 \times 10^3\right) \times \left(0.065^2 - 0.005^2\right)$$

~~u = 27.6225 m/s~~ $u = 27.628 m/s$

The shear stress τ can be found as:

$$\frac{\tau}{r} = \frac{\tau_o}{R}$$

$$\therefore \tau = \frac{r \times \tau_o}{R} = \frac{0.005 \times 342.0625}{0.065} = 26.3125 N/m^2$$