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Integrate the following with respect to their variable.

1. $3te^{2t}$

Solution

$$\int 3te^{2t}\,dt$$

$$3\int te^{2t}\,dt$$

Recall:

$$\int u\,dv = uv - \int v\,du$$

where:

$$u = t,\ dv = e^{2t}$$

$$du = dt,\ \int dv = \int e^{2t}$$

$$v = \frac{1}{2}e^{2t}$$

Substitute into the equation

$$= t \cdot \frac{1}{2}e^{2t} - \int \frac{1}{2}e^{2t} \cdot dt$$

$$= \frac{1}{2}te^{2t} - \frac{1}{2}\int e^{2t} \cdot dt$$

$$= \frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} + c$$

$$3\left(\frac{1}{2}te^{2t} - \frac{1}{4}e^{2t} + c\right) = \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + c$$

2. $x^2 \sin x$

Solution

$\int x^2 \sin x\, dx = uv - \int v\, du$

where:

$u = x^2$

$du = 2x\, dx$

$dv = \sin x$

$v = -\cos x$

$= -x^2 \sin x - \int -\cos x \cdot 2x\, dx$

$= -x^2 \cos x + \int 2x \cos x\, dx$

$= -x^2 \cos x + 2x \sin x - \int 2 \sin x\, dx$

$= -x^2 \cos x + 2x \sin x - (-2\cos x)$

$= -x^2 \cos x + 2x \sin x + 2\cos x + c$

3. $\sin 7x \cos 2x$

Solution

$\int \sin 7x \cos 2x$

Recall:

$\sin a \cdot \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$

$= \sin(7x)\cos(2x) = \frac{1}{2}\left(\sin(9x) + \sin(5x)\right)$

$= \frac{1}{2}\int \sin(9x)\, dx + \int \frac{1}{2} \sin(5x)\, dx$

$= \frac{1}{2}\left(\frac{-\cos(9x)}{9}\right) + \frac{1}{2}\left(\frac{-\cos(5x)}{5}\right) + c$

$= \frac{-\cos(9x)}{18} - \frac{\cos(5x)}{10} + c$

4) $\frac{(2x - 3x^2)}{1-x}$

Solution

$\int \frac{2x-3x^2}{1-x}\,dx$

let $u = 2x - 3x^2$

$du = 2 - 6x\,dx$

$dx = \frac{du}{2-6x}$

$\int \frac{u}{1-x} \times \frac{du}{2-6x}$

$\int \frac{u}{(1-x)(2-6x)}\,du = \int \frac{A}{(1-x)} + \frac{B}{(2-6x)}$

multiply through by $(1-x)(2-6x)$

$2x - 3x^2 = (2-6x)A + B(1-x)$

when $x = 1$: when $x = \frac{1}{3}$

$-1 = -\frac{1}{4}A$ $\quad \frac{1}{3} = \frac{2}{3}B$

$A = \frac{1}{4}$ $\quad B = \frac{1}{2}$

$\int 4(1-x) + \int 2(2-6x)$

$= -\frac{4}{2}(x) + -\frac{2}{2}(x) + c$

$= -2x - (x) + c$

$= -3x + c$