

Assignment

MAT 102

1) Find the equation of tangent at the point $[1,0]$ on the circle $x^2 + y^2 - 8x - 14y + 40 = 0$

Soln

To Finding the Centre of the circle.

$$x^2 + y^2 - 8x + 14y + 40 = 0$$

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Center } (h, k)$$

$$x^2 + y^2 - 8x + 14y + 40 = 0$$

$$x^2 - 8x + y^2 + 14y = -40$$

using Completing the squares we have 1.
 $\left[-\frac{8}{2}\right]^2 = (-4)^2 = 16$ $\left[\frac{14}{2}\right]^2 = (7)^2 = 49$

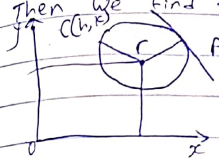
$$\therefore [x^2 - 8x + 16] + [y^2 + 14y + 49] = -40$$

$$[x-4]^2 + [y+7]^2 = -40 + 16 + 49$$

$$[x-4]^2 + [y+7]^2 = 25$$

\therefore Center $[4, 7]$

Then we find the gradient / slope of tangent -



$$P(x, y) = [1, 0]$$

$$C(h, k) = [4, 7]$$

$$\therefore M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 0}{4 - 1} = \frac{7}{3} \quad \therefore m = -\frac{3}{7}$$

\therefore Slope of tangent = $-\frac{3}{7}$ negative reciprocal of M.

Equation of tangent $y = mx + c$ $P(x, y)$

$$0 = -\frac{3}{7}(1) + c$$

$$c = \frac{3}{7}$$

$\therefore c = \frac{3}{7}$ $\therefore y = -\frac{3}{7}(x) + \frac{3}{7}$

2) Find the equation of tangent at the point $[1,0]$ on the circle $x^2 + y^2 - 12x - 12y + 47 = 0$

Soln

To Finding the Centre of the circle.

$$x^2 + y^2 - 12x - 12y + 47 = 0$$

$$(x-h)^2 + (y-k)^2 = r^2 \quad \text{Center } (h, k)$$

$$x^2 + y^2 - 12x - 12y + 47 = 0$$

$$x^2 - 12x + y^2 - 12y + 47 = 0$$

$$x^2 - 12x + y^2 - 12y = -47$$

Using Completing the squares we have:-

$$\left[-\frac{12}{2}\right]^2 = [-6]^2 = 36$$

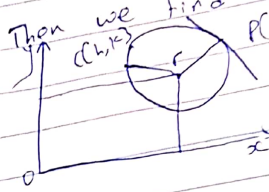
$$\left[-\frac{12}{2}\right]^2 = [-6]^2 = 36$$

$$1. [x^2 - 12x + 36] + [y^2 - 12y + 36] = -47$$

$$[x - 6]^2 + [y - 6]^2 = -47 + 36 + 36$$

$$[x - 6]^2 + [y - 6]^2 = 25$$

∴ Center $[6, 6]$



Then we find the gradient (slope of the tangent).

$$P(x, y) = [x, y]$$

$$C(h, k) = [6, 6]$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{6 - 1} = \frac{6}{5}$$

∴ Slope of tangent = $-\frac{5}{6}$ (negative reciprocal of m)

Find the Equation of tangent

$$y = mx + c$$

$$0 = -\frac{5}{6}(6) + c$$

$$\therefore c = \frac{5}{6}$$

$$1. y = mx + c$$

$$y = -\frac{5}{6}x + \frac{5}{6}$$

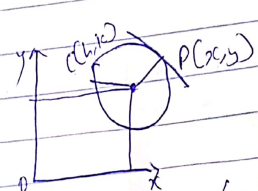
3) Find the equation of tangent at points $[1, 0]$ on circle

$$x^2 + y^2 - 5x - y + 4 = 0$$

Soln

$$C = [0, 0]$$

Center $[0, 0]$
 $P [1, 0]$



$$m = \frac{0 - 0}{1 - 0} = \frac{0}{1}$$

∴ Slope of tangent = -1

Equation of tangent

$$y = mx + c$$

$$0 = -1x + c \quad c = 1$$

$$\therefore 0 = -1x + 1$$