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Assignment

1) $x^2 \sin x \, dx$

Solution

$\int x^2 \sin x \, dx$

$u = x^2 \qquad dv = \sin x$

$du = 2x \, dx \qquad v = -\cos x$

$\int u\,dv = uv - \int v\,du$

$= x^2(-\cos x) - \int(-\cos x)(2x)\,dx$

$= -x^2 \cos x - \int -2x \cos x \, dx$

$\int -2x \cos x \, dx$

$u = -2x \qquad dv = \cos x$

$du = 2dx \qquad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$= (-2x)(\sin x) - \int(\sin x)(-2)dx$

$= -2x\sin x - \int -2\sin x \, dx$

$= -2x\sin x - (-2)\int \sin x \, dx$

$= -2x\sin x + 2\int \sin x \, dx$

$= -2x\sin x + 2(-\cos x)$

$= -2x \sin x - 2\cos x$

$\therefore \int x^2 \sin x \, dx = -x^2 \cos x - [-2x\sin x - 2\cos x]$

$= -x^2 \cos x + 2x\sin x + 2\cos x + C$

2) $3te^{2t}\,dt$

Solution

$\int 3te^{2t}\,dt$

$u = 3t \qquad dv = e^{2t}$

$du = 3\,dt \qquad v = \frac{1}{2}e^{2t}$

$\int u\,dv = uv - \int v\,du$

$= (3t)\left(\frac{1}{2}e^{2t}\right) - \int \frac{1}{2}e^{2t} \times 3\,dt$

$= 3t \times \frac{e^{2t}}{2} - \frac{3}{2}\int e^{2t}\,dt$

$= \frac{3te^{2t}}{2} - \frac{3}{2}\left[\frac{1}{2}e^{2t}\right]$

$= \frac{3te^{2t}}{2} - \frac{3}{4}e^{2t}$

$= \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$

3) $2x^2 \ln x \, dx$

Solution

$\int 2x^2 \ln x \, dx$

$2\int x^2 \ln x \, dx$

$u = \ln x \qquad dv = x^2$

$du = \frac{1}{x} \qquad v = \frac{x^3}{3}$

$\int u\,dv = uv - \int v\,du$

$= 2\left[\ln(x) \times \frac{x^3}{3} - \int \frac{x^3}{3} \times \frac{1}{x}\,dx\right.$

$= 2\left[\ln(x) \times \frac{x^3}{3} - \int \frac{x^2}{3}\,dx\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2\,dx\right]$

$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3} \times \frac{x^3}{3}\right] + C$

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$= 2\left[\frac{x^3\ln(x)}{3} - \frac{x^3}{9}\right] + c$

$= 2\left[= \frac{2x^3\ln(x)}{3} - \frac{2x^3}{9} + c\right.$

$= \left[\frac{2}{3}x^3\left(\ln(x) - \frac{1}{3}\right)\right] + c$

H2 $(2x - 3x^2/1 - x)dx$

Solution

$\int \frac{2x - 3x^2}{1-x} dx$

Separate the Fraction into 2 Fractions

$\int \frac{2x}{1-x} - \frac{3x^2}{1-x} dx$

$= \int \frac{2x}{1-x} dx - \int \frac{3x^2}{1-x} dx$

$= -2\left[\ln(1-x) - (1-x)\right]$

$= -2\left[\ln(1-x) - 1 + x\right]$

$\therefore \int \frac{2x}{1-x} dx = -2\ln(1-x) + 2 + 2x$

The second Fraction

$\int \frac{3x^2}{1-x} dx$

$3\int \frac{x^2}{1-x} dx$

$u = 1-x$

$\frac{du}{dx} = -1$

$du = -dx$

$dx = -du$

$\int \frac{3x^2}{1-x}\,dx = 3\int \frac{x^2}{u} - du = -3\int \frac{x^2}{u}\,du$

Recall, $u = 1-x$

$x = 1-u$

$= -3\int \frac{x^2}{u}\,du = -3\int \frac{(1-u)^2}{u}\,du = -3\int \frac{1-2u+u^2}{u}\,du$

$= -3\left[\int \frac{1}{u} - \frac{2u}{u} + \frac{u^2}{u}\,du\right] = -3\left[\int \frac{1}{u}\,du - \int \frac{2u}{u}\,du + \int \frac{u^2}{u}\,du\right]$

$= -3\left[\ln(u) - \int 2\,du + \int u\,du\right]$

$= -3\left[\ln(u) - 2u + \frac{u^2}{2}\right]$

$= -3\ln(u) + 6u - \frac{3u^2}{2}$

$= -3\ln(1-x) + 6(1-x) - \frac{3(1-x)^2}{2}$

$= -3\ln(1-x) + \frac{6(1-x)}{1} - \frac{3(1-2x+x^2)}{2}$

$= -3\ln(1-x + \frac{6\times 2(1-x) - 3(1-2x+x^2)}{2}$

$= -3\ln(1-x) + \frac{12-12x-3+6x-3x^2}{2}$

$= -3\ln(1-x) + \frac{9-6x-3x^2}{2}$

So,

$\int \frac{2x}{x-x}\,dx - \int \frac{3x^2}{1-x}\,dx = -2\ln(1-x)+2-2x - \left[-3\ln(1-x)+\frac{9-6x-3x^2}{2}\right]$

$= -2\ln(1-x)+2-2x+3\ln(1-x) - \frac{9+6x+3x^2}{2} + C$

$= \frac{2-2x}{1} - \frac{9+6x+3x^2}{2} - 2\ln(1-x) + 3\ln(1-x)$

$$= \frac{4 - 4x - 9 + 6x + 3x^2}{2} + \ln(1-x) + c$$

$$\int \frac{2x - 3x^2}{(1-x)}\,dx = \frac{3x^2 + 2x - 5}{2} + \ln(1-x) + c$$