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Assignment.

(1.) $x^2 \sin x \, dx$

Solution.

$$\int x^2 \sin x \, dx$$

$u = x^2$

$du = 2x \, dx$ $\quad dv = \sin x$

$v = -\cos x$

$$\int u \, dv = uv - \int v \, du$$

$$= x^2(-\cos x) - \int (-\cos x)(2x) \, dx$$

$$= -x^2 \cos x - \int -2x \cos x \, dx$$

$$\int -2x \cos x \, dx$$

$u = -2x$ $\quad dv = \cos x$

$du = -2 \, dx$ $\quad v = \sin x$

$$\int u \, dv = uv - \int v \, du$$

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$= (-2x)(\sin x) - \int (\sin x)(-2)\,dx$

$= -2x\sin x - \int (-2\sin x)\,dx$

$= -2x\sin x - (-2)\int \sin x\,dx$

$= -2x\sin x + 2\int \sin x\,dx$

$= -2x\sin x + 2(-\cos x)$

$= -2x\sin x - 2\cos x.$

$\therefore \int x^2 \sin x\,dx = -x^2\cos x - \left[-2x\sin x - 2\cos x\right]$

$= -x^2\cos x + 2x\sin x + 2\cos x + c$

(2) $3te^{2t}\,dt$

Solution

$\int 3te^{2t}\,dt$

$u = 3t$

$du = 3\,dt$ $\quad$ $dv = e^{2t}$

$v = \frac{1}{2}e^{2t}$

$\int u\,dv = uv - \int v\,du$

$= (3t)\left(\frac{1}{2}e^{2t}\right) - \int \frac{1}{2}e^{2t} \times 3\,dt$

$$= 3t \times \frac{e^{2t}}{2} - \frac{3}{2}\int e^{2t}\,dt$$

$$= \frac{3te^{2t}}{2} - \frac{2}{3}\left[\frac{1}{2}e^{2t}\right]$$

$$= \frac{3te^{2t}}{2} - \frac{3}{4}e^{2t}$$

$$= \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

(3) $2x^2 \ln x \, dx$

$$2\int x^2 \ln x \, dx$$

$u = \ln x \quad dv = x^2$

$du = \frac{1}{x} \quad v = \frac{x^3}{3}$

$$\int u\,dv = uv - \int v\,du$$

$$= 2\left[\ln(x) \times \frac{x^3}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x}\,dx\right]$$

$$= 2\left[\ln(x) \times \frac{x^3}{3} - \int \frac{x^2}{3}\,dx\right]$$

$$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3}\int x^2\,dx\right]$$

$$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{1}{3} \times \frac{x^3}{3}\right] + C$$

$$= 2\left[\frac{x^3 \ln(x)}{3} - \frac{x^3}{9}\right] + C$$

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$$= \frac{2x^3 \ln(x)}{3} - \frac{2x^3}{9} + c$$

$$= \left[\frac{2}{3}x^3\left(\ln(x) - \frac{1}{3}\right)\right] + c$$

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4) $(2x-3x^2)/_{(1-x)}\, dx$

Solution

$$\int \frac{2x-3x^2}{1-x}\, dx$$

Separate the fraction into 2 fractions

$$\int \frac{2x}{1-x} - \frac{3x^2}{1-x}\, dx$$

$$= \int \frac{2x}{1-x}\, dx - \int \frac{3x^2}{1-x}\, dx$$

The first fraction

$$\int \frac{2x}{1-x}\, dx$$

$u = 1-x$

$\frac{du}{dx} = -1$

$\frac{du}{-1} = \frac{\cancel{-}dx}{\cancel{-}1}$

$dx = -du$

$$\int \frac{2x}{1-x}\, dx = \int \frac{2x}{u}\, dx = \int \frac{2x}{u} \cdot -du = -2\int \frac{x}{u}\, du$$

Recall, $u = 1-x$

$x = 1-u$

$$= -2\int \frac{x}{u}\, du = -2\int \frac{(1-u)}{u}\, du$$

$$= -2\left[\int \frac{1}{u} - \frac{u}{u}\, du\right] = -2\left[\int \frac{1}{u}\, du - \int \frac{\cancel{u}}{\cancel{u}}\, du\right]$$

$$= -2\left[\ln(u) - \int 1\, du\right]$$

$$= -2\left[\ln(u) - u\right]$$

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$$= -2\left[\ln(1-x) - (1-x)\right]$$
$$= -2\left[\ln(1-x) - 1 + x\right]$$
$$\therefore \int \frac{2x}{1-x}\,dx = -2\ln(1-x) + 2 - 2x$$

The second fraction

$$\int \frac{3x^2}{1-x}\,dx$$

$$3\int \frac{x^2}{1-x}\,dx$$

$u = 1-x$

$\frac{du}{dx} = -1$

$du = -dx$

$dx = -du$

$$\int \frac{3x^2}{1-x}\,dx = 3\int \frac{x^2}{u} \cdot -du = -3\int \frac{x^2}{u}\,du$$

Recall, $u = 1-x$

$x = 1-u$

$$= -3\int \frac{x^2}{u}\,du = -3\int \frac{(1-u)^2}{u}\,du = -3\int \frac{1-2u+u^2}{u}\,du$$

$$= -3\left[\int \frac{1}{u} - \frac{2u}{u} + \frac{u^2}{u}\,du\right] = -3\left[\int \frac{1}{u}\,du - \int \frac{2\cancel{u}}{\cancel{u}}\,du + \int \frac{u^{\cancel{2}}}{\cancel{u}}\,du\right]$$

$$= -3\left[\ln(u) - \int 2\,du + \int u\,du\right]$$

$$= -3\left[\ln(u) - 2u + \frac{u^2}{2}\right]$$

$$= -3\ln(u) + 6u - \frac{3u^2}{2}$$

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$$= -3\ln(1-x) + 6(1-x) - \frac{3(1-x)^2}{2}$$

$$= -3\ln(1-x) + \frac{6(1-x)}{1} - \frac{3(1-2x+x^2)}{2}$$

$$= -3\ln(1-x) + \frac{(6\times2)(1-x) - 3(1-2x+x^2)}{2}$$

$$= -3\ln(1-x) + \frac{12(1-x) - (3-6x+3x^2)}{2}$$

$$= -3\ln(1-x) + \frac{12-12x-3+6x-3x^2}{2}$$

$$\int\frac{3x^2}{1-x}\,dx = -3\ln(1-x) + \frac{9-6x-3x^2}{2}$$

4

So,

$$\int\frac{2x}{1-x}\,dx - \int\frac{3x^2}{1-x}\,dx = -2\ln(1-x)+2-2x-\left[-3\ln(1-x)+\frac{9-6x-3x^2}{2}\right]$$

$$= -2\ln(1-x)+2-2x+3\ln(1-x)-\frac{9+6x+3x^2}{2}$$

$$= \frac{2-2x}{1} - \frac{9+6x+3x^2}{2} - 2\ln(1-x)+3\ln(1-x)$$

$$= \frac{2(2-2x)-9+6x+3x^2}{2} + \ln(1-x) + C$$

$$= \frac{4-4x-9+6x+3x^2}{2} + \ln(1-x) + C$$

$$\int\frac{2x-3x^2}{(1-x)}\,dx = \frac{3x^2+2x-5}{2} + \ln(1-x) + C$$

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