

As in DE, Jee/

18/04/02) 016

Camp Eng

$$\begin{aligned} T_0 &= 10^\circ\text{C} \\ T_1 &= 20^\circ\text{C} \\ T_{\text{ambient}} &= 25^\circ\text{C} \end{aligned}$$

① Solving
 $\frac{dT}{dt} \propto (T - T_0)$ ($T_0 = \text{ambient temp}$)

$$\frac{dT}{dt} = k(T - T_0)$$

$$\frac{dT}{T - 25} = k(T - 25)$$

Cooling law + const

$$\frac{dT}{T - 25} = k dt$$

$$(T - 25)$$

Integrate both sides

$$\ln(T - 25) = kt + C$$

$$T - 25 = e^{kt+C} \quad \text{where } e^C = A$$

$$T - 25 = e^{kt} \cdot e^C$$

$$T - 25 = A e^{kt}$$

$$T = A e^{kt} + 25$$

at initial condition $t=0$ $T=10^\circ\text{C}$

$$10 = A e^{0} + 25$$

$$T = 35 e^{kt} + 25$$

at $T=20^\circ\text{C}$ $t=5 \text{ mins}$

$$20 = 35 e^{5k} + 25$$

$$-5 = 35 e^{5k}$$

$$e^{5k} = -5/35$$

$$5k = \ln(-5/35)$$

$$k = \frac{\ln(-5/35)}{5} \quad k = 0.05 \text{ s}^{-1}$$

$$T = 35e^{0.05t} - 25$$

$$T = 29.9 \text{ at } t = ?$$

$$29.9 = 35e^{0.05t} - 25$$

$$49.9 = 35e^{0.05t}$$

$$e^{0.05t} = 49.9/35$$

$$e^{0.05t} = \ln(1.426)$$

$$0.05t = 0.355$$

$$t = 7.1 \text{ minutes}$$