Obot Mkpouto Obot
A/EN/G04/046
Elect / Elect
Circuit theory assignment

(1) $\omega_0, \omega_1, \omega_2, Q_2$ and B

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20\times10^{-3})(5\times10^{-9})}} = 100000$$

$$= 100 \text{ krad/s}$$

$$Q = \frac{R}{\omega_0 L} = \frac{100000}{(100000 \times 20\times10^{-3})} = 50$$

Since $Q >> 10$, the circuit is a high Q circuit hence

$$\omega_1 \simeq \omega_0 - B/2 \qquad \omega_2 \simeq \omega_0 + B/2$$

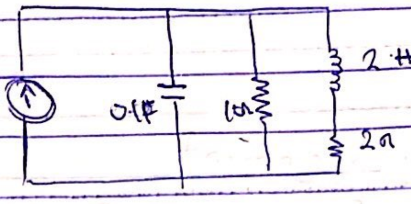
$$B = \frac{\omega_0}{Q} = \frac{100000}{50} = 2000 = 2\text{k rad/s}$$

$$\omega_1 \simeq 100000 - \frac{2000}{2} = 99000 = 99\text{krad/s}$$

$$\omega_2 \simeq 100000 - \frac{2000}{2} = 101000 = 10.1 \text{ krad/s}$$

ii) $$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\;}}$$

2)



$$Y = \frac{1}{10} + j\omega \cdot 0.1 + \frac{1}{2+j\omega 2} = 0.1 + j\omega 0.1 + \frac{1}{2+j\omega 2}$$

rationalizing $\frac{1}{2+j\omega 2} = \frac{1}{2+j\omega 2} \times \frac{2-j\omega 2}{2-j\omega 2} = \frac{2-j\omega 2}{4-4j\omega+4j\omega+4\omega^2}$

$$= \frac{2-j\omega 2}{4+4\omega^2}$$

$$Y = 0.1 + j\omega 0.1 + \frac{2-j\omega 2}{4+4\omega^2} = 0.1 + j\omega 0.1 + \frac{2}{4+4\omega} - \frac{j\omega^2}{4+4\omega^2}$$

$$Y = 0.1 + \frac{2}{4+4\omega^2} + j\omega 0.1 - \frac{j\omega^2}{4+4\omega^2} = 0.1 + \frac{2}{4+4\omega^2} + j\left(\omega 0.1 - \frac{\omega_o^2}{4+4\omega^2}\right)$$

At resonance $Im(Y) = 0$

Therefore, $\omega\, 0.1 - \frac{\omega_o^2}{4+4\omega^2} = 0$

$$0.1 - \frac{2}{4+4\omega^2} = 0$$

$$0.1 = \frac{2}{4+4\omega_o^2} \qquad 0.4 + 0.4\omega_o^2 = 2$$

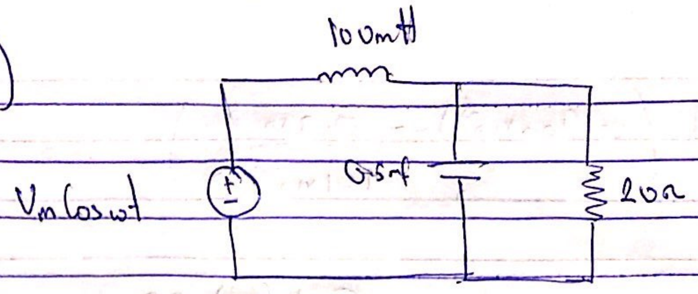
$$0.4\omega_o^2 = 1.6$$

$$\omega_o^2 = 4 \qquad \sqrt{\omega_o^2} = \sqrt{4}$$

$\omega_o = 2$

$\omega_o = 2$ rad/s

3)



$$Z = \left(j\omega\, 100\times10^{-3} + \left(\frac{20}{j\omega(0.5\times10^{-3})} \Big/ \frac{20}{1} + \frac{1}{0.5\times10^{-3} j\omega} \right) \right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \Big/ \left(\frac{0.01 j\omega + 1}{0.5\times10^{-3} j\omega} \right) \right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \times \frac{j\omega(0.5\times10^{-3})}{0.01 j\omega + 1} \right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{0.01 j\omega + 1} \right)$$

rationalising $\frac{20}{0.01j\omega+1} = \frac{20}{0.01j\omega+1} \times \frac{0.01j\omega-1}{0.01j\omega-1}$

$$= \frac{j\omega 0.2 - 20}{-1\times10^{-4}\omega^2 - j\omega 0.01 + j\omega 0.01 - 1}$$

$$= \frac{j\omega 0.2 - 20}{-1\times10^{-4}\omega^2 - 1} = \frac{-20 + j\omega 0.2}{-1\times10^{-4}\omega^2 - 1}$$

$$= \frac{-(20 - 0.2j\omega)}{-(1\times10^{-4}\omega^2 + 1)} = \frac{20 - 0.2j\omega}{1 + 1\times10^{-4}\omega^2} = \frac{20}{1+1\times10^{-4}\omega^2} + j\omega \frac{0.2}{1+1\times10^{-4}\omega^2}$$

$$Z = j\omega(100\times10^{-3}) + \frac{20}{1+1\times10^{-4}\omega^2} - \frac{0.2j\omega}{1+1\times10^{-4}\omega^2} =$$

$$Z = \frac{20}{1+1\times10^{-4}\omega^2} + j\omega(100\times10^{-3}) - \frac{0.2j\omega}{1+1\times10^{-4}\omega^2}$$

$$Z = \frac{20}{1+1\times10^{-4}\omega^2} + j\left(\omega(100\times10^{-3}) - \frac{0.2\omega}{1+1\times10^{-4}\omega^2}\right)$$

At resonance the Imaginary part of Z, Img (z') = 0

$$= \omega_0(100\times10^{-3}) - \frac{0.2\omega_0}{1+1\times10^{-4}\omega_0^2} = 0$$

$$\omega_0(100\times10^{-3}) = \frac{0.2\omega_0}{1+1\times10^{-4}\omega_0^2}$$

Cross multiply

$$\omega_0(100\times10^{-3})(1+1\times10^{-4}\omega_0^2) = 0.2\omega_0$$

$$= 0.1 + 1\times10^{-5}\omega_0^2 = 0.2$$

$$= 1\times10^{-5}\omega_0^2 = 0.2 - 0.1$$

$$\omega_0^2 = \frac{0.1}{1\times10^{-5}}$$

$$\omega_0^2 = 10000$$

$$\omega_0 = \sqrt{10000}$$

$$\omega_0 = 100 rad/s$$