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18/ENG04/082
Circuit Theory

A parallel resonant circuit has $R = 100\,k\Omega$, $L = 20\,mH$ & $C = 5nF$. Calculate $\omega_0, \omega_1, \omega_2$ & B

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20\times10^{-3})(5\times10^{-9})}} = 100\,000\,rad/s$$

$$\omega_0 = 100\,krad/s$$

$$Q = \frac{R}{\omega_0 L} = \frac{100\times10^3}{(100\,000)(20\times10^{-3})} = \frac{1}{20\times10^{-3}} = 50$$

$$B = \frac{\omega_0}{Q} = \frac{100\times10^3}{50} = 2000 = 2\,krad/s$$

Since $Q > 1.0$, $\omega_1 = \omega_0 - \frac{B}{2} = 100\times10^3 - \frac{2000}{2}$

$$= 99000$$

$$\omega_1 = 99\,krad/s$$

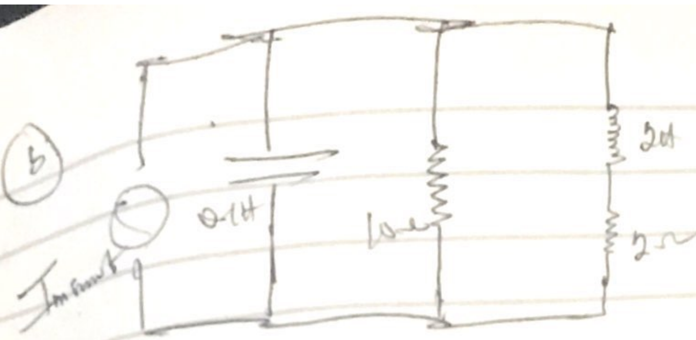
$$\omega_2 = \omega_0 + \frac{B_0}{2} = 100\times10^3 + \frac{2000}{2}$$

$$= 100\times10^3 + 1000$$

$$= 101000$$

$$\omega_2 = 1001\,krad/s$$

(b)



$$Y = \frac{1}{10} + j\omega 0.1 + \frac{1}{2+j\omega 2} = 0.1 + j\omega 0.1 + \frac{1}{2+j\omega 2}$$

Rationalizing

$$\frac{1}{2+j\omega 2} = \frac{1}{2+j\omega 2} \times \frac{2-j\omega 2}{2-j\omega 2} = \frac{2-j\omega 2}{4-4j\omega+4j\omega+4\omega^2}$$

$$= \frac{2-j\omega 2}{4+4\omega^2}$$

$$Y = 0.1 + j\omega 0.1 + \frac{2-j\omega 2}{4+4\omega^2} = 0.1 + j\omega 0.1 + \frac{2}{4+4\omega} - \frac{j\omega 2}{4+4\omega^2}$$

$$Y = 0.1 + \frac{2}{4+4\omega^2} + j\omega 0.1 - \frac{j\omega 2}{4+4\omega^2} = 0.1 + \frac{2}{4+4\omega^2} + j\left(\omega 0.1 - \frac{\omega 2}{4+4\omega^2}\right)$$

At resonance $I = (Y) = 0$

$$\omega_0 0.1 - \frac{\omega_0 2}{4+4\omega^2} = 0$$

$$0.1 - \frac{2}{4+4\omega^2} = 0$$

$$0.1 = \frac{2}{4+4\omega^2}$$

$$0.4 + 0.4\omega^2 = 2$$

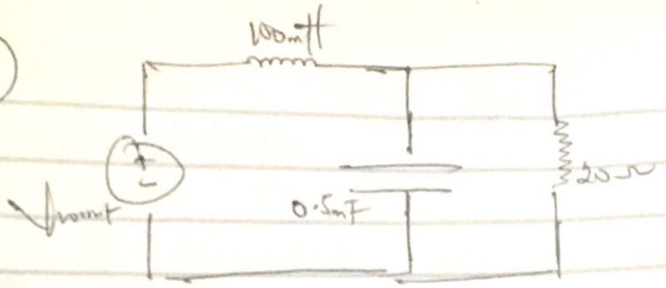
$$0.4\omega^2 = 1.6$$

$$\omega_0^2 = 4$$

$$\omega_0 = 2$$

$$\omega_0 = 2 \text{ rad/s}$$

(c)



$$Z = j\omega 100\times10^{-3} + \left(\frac{20}{j\omega(0.5\times10^{-3})} \div \left(\frac{20}{1} + \frac{1}{0.5\times10^{-3}j\omega}\right)\right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \div \left(\frac{0.01j\omega+1}{0.5\times10^{-3}j\omega}\right)\right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \times \frac{j\omega(0.5\times10^{-3})}{0.01j\omega+1}\right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{0.01j\omega+1}\right)$$

Rationalising $\frac{20}{0.01j\omega+1} = \frac{20}{0.01j\omega+1} \times \frac{0.01j\omega-1}{0.01j\omega-1}$

$$= \frac{j\omega0.2 - 20}{-1\times10^{-4}\omega^2 - j\omega0.01 + j\omega0.01 - 1}$$

$$= \frac{j\omega0.2-20}{-1\times10^{-4}\omega^2-1} = \frac{-20+j\omega0.2}{-1\times10^{-4}\omega^2-1}$$

$$= \frac{-(20-0.2j\omega)}{-(1\times10^{-4}\omega^2+1)} = \frac{20-0.2j\omega}{1+1\times10^{-4}\omega^2} = \frac{20}{1+1\times10^{-4}\omega^2} + j\omega\frac{0.2}{1+1\times10^{-4}\omega^2}$$

$$Z = \frac{20}{1+1\times10^{-4}\omega^2} + j\omega(100\times10^{-3}) - \frac{0.2j\omega}{1+1\times10^{-4}\omega^2}$$

$$Z = \frac{20}{1+1\times10^{-4}\omega^2} + j\left(\omega(100\times10^{-3}) - \frac{0.2\omega}{1+1\times10^{-4}\omega^2}\right)$$

At resonance

$$\omega_0(100\times10^{-3}) - \frac{0.2\omega_0}{1+1\times10^{-4}\omega_0^2} = 0$$

$$\omega_0(100\times10^{-3}) = \frac{0.2\omega_0}{1+1\times10^{-4}\omega_0^2}$$

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$$\omega_0 (100 \times 10^{-3})(1 + 1 \times 10^{-4} \omega_0) = 0.2$$

$$= 1 \times 10^{-5} \omega_0^2 = 0.2 - 0.1$$

$$\omega_0^2 = \frac{0.1}{1 \times 10^{-5}}$$

$$\omega_0^2 = 10000$$

$$\omega_0 = \sqrt{10000}$$

$$\omega_0 = 100 \text{ rad/s}$$