

IKPEAMA JOHN

17/ENG04/031

ENG 322

ELECT / ELECT

1. A Parallel resonant circuit has $R = 100\text{ k}\Omega$, $L = 20\text{ mH}$, and $C = 5\text{ mF}$. Calculate ω_0 , ω_1 , ω_2 , Q and B .

Solution

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20 \times 10^{-3}) \times (5 \times 10^{-3})}}$$
$$= 100 \text{ krad/s}^{-1}$$

$$B = \frac{\omega_0}{Q} = \frac{1}{RC} = \frac{1}{100 \times (5 \times 10^{-3})}$$
$$= 2 \text{ krad/s}$$

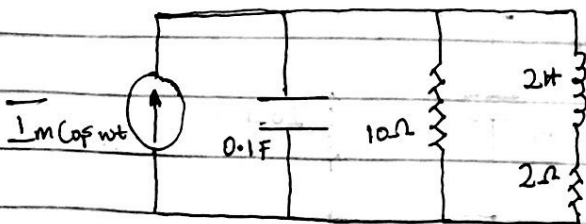
$$\therefore Q = \frac{\omega_0}{B} = \frac{100 \times 10^3}{2 \times 10^3} = 50$$

Since $Q \geq 10$

$$\omega_1 = \omega_0 - \left(\frac{B}{2}\right) = 100 \times 10^3 - \frac{2 \times 10^3}{2} = 99 \text{ krad/s}$$

$$\omega_2 = \omega_0 + \left(\frac{B}{2}\right) = 100 \times 10^3 + \frac{2 \times 10^3}{2} = 101 \text{ krad/s}$$

2. Determine the resonant frequency of the circuit below



The input admittance is

$$Y = j\omega \cdot 1 + \frac{1}{10} + \frac{1}{2 + j\omega 2}$$

$$= 0.1 + j\omega \cdot 1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

at resonance.

$$\text{Im}(Y) = 0$$

$$\frac{\omega \cdot 0.1 - \omega \cdot 2}{4 + 4\omega^2} = 0$$

$$\frac{0.1 - 2}{4 + 4\omega_0^2} = 0$$

$$0.1 = \frac{2}{4 + 4\omega_0^2}$$

$$0.4 + 0.4\omega_0^2 = 2$$

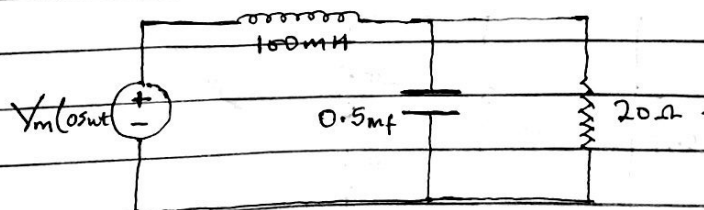
$$0.4\omega_0^2 = 1.6$$

$$\omega_0^2 = 4$$

$$\omega_0 = \sqrt{4}$$

$$\omega_0 = 2 \text{ rad/s}$$

3. Calculate the resonant frequency of the circuit below



Solution

$$Z = j\omega 100 \times 10^{-3} + \left(\frac{20}{j\omega(0.5 \times 10^{-3})} \right) \div \left(\frac{20}{1} + \frac{1}{0.5 \times 10^{-3} j\omega} \right)$$

$$Z = j\omega(100 \times 10^{-3}) + \left(\frac{20}{j\omega(0.5 \times 10^{-3})} \div \left(\frac{0.01j\omega + 1}{0.5 \times 10^{-3} j\omega} \right) \right)$$

$$Z = j\omega(100 \times 10^{-3}) + \left(\frac{20}{j\omega(0.5 \times 10^{-3})} \times \frac{0.5 \times 10^{-3} j\omega}{0.01j\omega + 1} \right)$$

$$Z = j\omega(100 \times 10^{-3}) + \left(\frac{20}{0.01j\omega + 1} \right)$$

Conjugating $\frac{20}{0.01j\omega + 1} = \frac{20}{0.01j\omega + 1} \times \frac{0.01j\omega - 1}{0.01j\omega - 1}$

$$Z = \frac{j\omega 0.2 - 20}{-1 \times 10^{-4} \omega^2 - j\omega 0.01 + j\omega 0.01}$$

$$Z = \frac{-20 + j\omega 0.2}{1 \times 10^{-4} \omega^2 + 1}$$

$$Z = \frac{20 - 0.2j\omega}{1 + 1 \times 10^{-4} \omega^2}$$

$$Z = j\omega(100 \times 10^{-3}) + \frac{20}{1 + 1 \times 10^{-4} \omega^2} - \frac{0.2j\omega}{1 + 1 \times 10^{-4} \omega^2}$$

$$Z = \frac{20}{1 + 1 \times 10^{-4} \omega^2} + j\omega(100 \times 10^{-3}) - \frac{0.2j\omega}{1 + 1 \times 10^{-4} \omega^2}$$

$$Z = \frac{20}{1 + 1 \times 10^{-4} \omega^2} + j \left(\frac{\omega(100 \times 10^{-3}) - 0.2\omega}{1 + 1 \times 10^{-4} \omega^2} \right)$$

At resonance, the imaginary side of $Z_{in}(s) = 0$

$$Z_0 (100 \times 10^{-3}) - \frac{0.2 Z_0}{1 + 1 \times 10^{-4} Z_0^2} = 0$$

$$Z_0 (100 \times 10^{-3}) = \frac{0.2 Z_0}{1 + 1 \times 10^{-4} Z_0^2}$$

$$\cancel{Z_0} (100 \times 10^{-3}) (1 + 1 \times 10^{-4} Z_0) = 0.2 \cancel{Z_0}$$
$$= 0.1 + 1 \times 10^{-5} Z_0^2 = 0.2$$

$$= 1 \times 10^{-5} Z_0^2 = 0.2 - 0.1$$

$$Z_0^2 = \frac{0.1}{1 \times 10^{-5}}$$

$$= 10000$$

$$Z_0 = 100 \text{ rad/s}$$