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COURSE: MATH 104
MATRIC NO: 19/ENG09/013
DEPARTMENT: AERONAUTICAL ENGINEERING
DATE: 28/04/2020

1. $\frac{4x^2 - \sin x}{x^3}$ as $x \to 0$

$$= \lim_{x \to 0} \left\{ \frac{4x^2 - \sin x}{x^3} \right\} = \lim_{x \to 0} \left\{ \frac{8x - \cos x}{3x^2} \right\} = \lim_{x \to 0} \left\{ \frac{8 + \sin x}{6x} \right\}$$

$$= \lim_{x \to 0} \left\{ \frac{\cos x}{6} \right\} = \underline{\underline{1/6}}$$

2. $\frac{dy}{dx}$ of $\frac{7x^2 \cos 8x}{e^{3x}}$

$$\frac{dy}{dx} = y \left(\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx} \right)$$

$u = 7x^2$, $\frac{du}{dx} = 14x$

$v = \cos 8x$, $\frac{dv}{dx} = -8\sin 8x$

$w = e^{3x}$, $\frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y \left(\frac{1}{7x^2}(14x) + \frac{1}{\cos 8x}(-8\sin 8x) - \frac{1}{e^{3x}}(3e^{3x}) \right)$$

$$\frac{dy}{dx} = y \left(2/x - 8\tan 8x - 3 \right)$$

$$\therefore \frac{dy}{dx} = \left(\frac{7x^2 \cos 8x}{e^{3x}} \right) \left(2/x - 8\tan 8x - 3 \right)$$

3. $\frac{dy}{dx}$ of $\cos(5x^2+6x)$

let $u = 5x^2+6x$ $\therefore$ $y = \cos u$

$\frac{du}{dx} = 10x+6$, $\frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= -\sin u \times (10x+6)$

$\therefore \frac{dy}{dx} = -(10x+6) \sin(5x^2+6x)$

$\frac{dy}{dx} = -(10x+6) \sin(5x^2+6x)$

4. $\int \frac{3}{4x+1} dx$

$= 3 \int \frac{1}{4x+1} dx$

let $4x+1 = u$

$\frac{du}{dx} = 4$

$dx = \frac{du}{4}$

$= 3 \int \frac{1}{u} \cdot \frac{du}{4}$

$= \frac{3}{4} \int \frac{1}{u} \cdot du$

$= \frac{3}{4} \ln u + C$

$= \frac{3}{4} \ln(4x+1) + C$

4b. $\int \frac{dx}{x^2+49} = \int \frac{dx}{x^2+7^2}$

Let $x = 7\tan\theta$ $\quad \frac{dx}{d\theta} = 7\sec^2\theta$, $dx = 7\sec^2\theta \, d\theta$

$$\int \frac{7\sec^2\theta \, d\theta}{(7\tan\theta)^2+49} = \int \frac{7\sec^2\theta}{49\tan^2\theta+49} d\theta$$

$$= \int \frac{7\sec^2\theta}{49(\tan^2\theta+1)} d\theta = \int \frac{\cancel{7}^{1}\cancel{\sec^2\theta}}{\cancel{49}_{7}\cancel{\sec^2\theta}} d\theta$$

$$= \frac{1}{7}\int d\theta = \frac{1}{7}\cdot\theta$$

But $\theta = \frac{x}{7}\tan^{-1}$

$$\therefore \int \frac{dx}{x^2+49} = \frac{1}{7}\tan^{-1}\frac{x}{7} + C$$

4c. $\int e^{6x} + 9x^3 - \sin 7x + \cos 8x \, dx$

$$= \int e^{6x} dx + \int 9x^3 dx - \int \sin 7x \, dx + \int \cos 8x \, dx$$

$$= \frac{1}{6}e^{6x} + \frac{9x^4}{4} - \frac{1}{7}(-\cos 7x) + \frac{1}{8}\sin 8x + C$$

$$= \frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$$

4d. $\int x\sqrt{9+x^2}\,dx$

Let $9+x^2=u$

$\therefore \frac{du}{dx}=2x$, $dx=\frac{du}{2x}$

$\int \cancel{x}\,u^{1/2}.\frac{du}{2\cancel{x}}$

$\frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}\left[\frac{u^{1/2+1}}{1/2+1}\right]+C$

$=\frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]+C$

$=\frac{1}{3}u^{3/2}+C$

$=\frac{1}{3}(9+x^2)^{3/2}+C$