NAME: OLUTAYO AYODEJI SAMUEL
Matric No: 171ENG05103+
Department: Mechatronics Engineering
Course: EMT 322

A parallel resonant circuit has $R = 100k\Omega$, $L = 20mH$ and $C = 5\mu F$. Calculate $\omega_0, \omega_1, \omega_2, Q$ and B

$R = 100K\Omega$, $L = 20mH$, $C = 5\mu F$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20\times10^{-3})(5\times10^{-9})}} = \frac{1}{\sqrt{1\times10^{-10}}} = 100000 rad/s$$

$\omega_0 = 100k rad/s$

$$Q = \frac{R}{\omega_0 L} = \frac{100\times10^3}{(100\times10^3).(20\times10^{-3})} = \frac{1}{20\times10^{-3}} = 50$$

$Q = 50$

$$B = \frac{\omega_0}{Q} = \frac{100\times10^3}{50} = 2000 = 2k rad/s.$$

Since $Q > 1.0$, $\omega_1 = \omega_0 - \frac{B}{2} = 100\times10^3 - \frac{2000}{2}$

$= 100\times10^{-3} - 1000$

$= 99000$

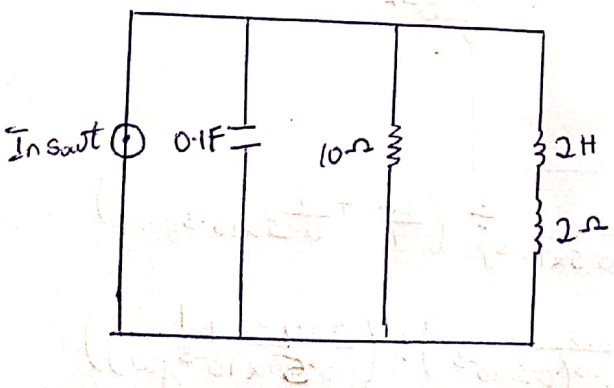
$\omega_1 = 99k rad/sec$

$$\omega_2 = \omega_0 + \frac{B_0}{2} = 106\times10^3 + \frac{2000}{2}$$

$= 100\times10^3 + 1000$

$= 101000$

$\omega_2 = 101k rad/s$



$$Y = \frac{1}{10} + j\omega 0.1 + \frac{1}{2+j\omega 2} = 0.1 + j\omega 0.1 + \frac{1}{2+j\omega 2}$$

Rationalizing $\frac{1}{2+j\omega 2} = \frac{1}{2+j\omega_2} \times \frac{2-j\omega 2}{2-j\omega 2} = \frac{2-j\omega^2}{4-4j\omega+4j\omega+4\omega^2}$

$$= \frac{2-j\omega 2}{4+4\omega^2}$$

$$Y = 0.1 + j\omega 0.1 + \frac{2 - j\omega 2}{4+4\omega^2} = 0.1 + j\omega 0.1 + \frac{2}{4+4\omega} = \frac{j\omega^2}{4+4\omega}$$

$$Y = 0.1 + \frac{2}{4+4\omega^2} + j\omega 0.1 - \frac{j\omega^2}{4+4\omega^2} = 0.1 + \frac{2}{4+4\omega^2} + j\left(\omega 0.1 - \frac{\omega 2}{4+4\omega^2}\right)$$

At resonance, $Im(Y) = 0$

Therefore, $\omega_0 0.1 - \frac{\omega_0 2}{4+4\omega_0^2} = 0$

$$0.1 - \frac{2}{4+4\omega_0^2} = 0$$

$$0.1 = \frac{2}{4+4\omega_0^2}$$

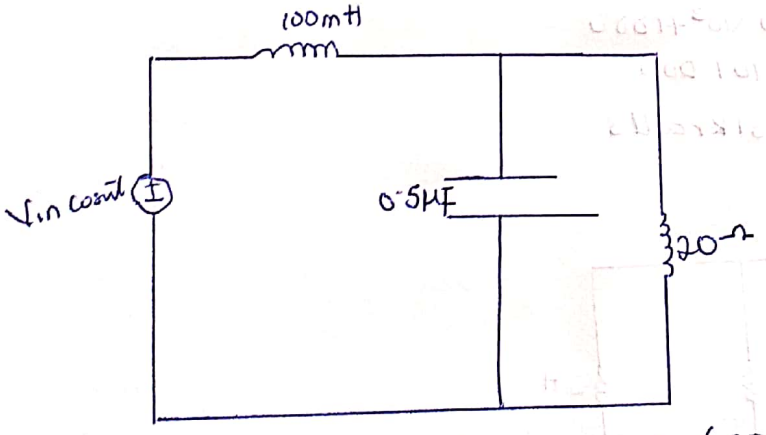
$$0.4 + 0.4\omega_0^2 = 2$$

$$0.4\omega_0^2 = 1.6$$

$$\omega_0^2 = 4$$

$$\omega_0 = \sqrt{4}$$

$$\omega_0 = 2 \text{ rad/s}$$



$$Z = j\omega 100 \times 10^{-3} + \left(\frac{20}{j\omega(0.5\times10^{-3})} \div \left(\frac{20}{1} + \frac{1}{0.5\times10^{-3} j\omega}\right)\right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \div \left(\frac{0.01 j\omega + 1}{0.5\times10^{-3} j\omega}\right)\right)$$

$$j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \times \frac{j\omega(0.5\times10^{-3})}{0.01 j\omega + 1}\right)$$

$$j\omega(100\times10^{-3}) + \left(\frac{20}{0.01 j\omega + 1}\right)$$

Rationalising $\frac{20}{0.01 j\omega + 1} = \frac{20}{0.01 j\omega + 1} \times \frac{0.01 j\omega - 1}{0.01 j\omega - 1}$

$$= \frac{j\omega 0.2 - 20}{-1\times10^{-4}\omega^2 - j\omega 0.01 + j\omega 0.01 - 1}$$

$$= \frac{j\omega 0.2 - 20}{-1\times10^{-4}\omega^2 - 1} \qquad = \frac{-20 + j\omega 0.2}{-1\times10^{-4}\omega^2 - 1}$$

$$= \frac{-(20 - 0.2j\omega)}{-(1\times10^{-4}\omega^2 + 1)} = \frac{20 - 0.2j\omega}{1 + 1\times10^{-4}\omega^2} = \frac{20}{1 + 1\times10^{-4}\omega^2} + \frac{j\omega 0.2}{1 + 1\times10^{-4}\omega^2}$$

$$Z = j\omega(100\times10^{-3}) + \frac{20}{1 + 1\times10^{-4}\omega^2} = \frac{0.2j\omega}{1 + 1\times10^{-4}\omega^2}$$

$$Z = \frac{20}{1 + 1\times10^{-4}\omega^2} + j\omega(100\times10^{-3}) - \frac{0.2j\omega}{1 + 1\times10^{-4}\omega^2}$$

$$Z = \frac{20}{1 + 1\times10^{-4}\omega^2} + j\left(\omega(100\times10^{-3}) - \frac{0.2\omega}{1 + 1\times10^{-4}\omega^2}\right)$$

At resonance the imaginary part of Z, $\mathrm{Im}(Z) = 0$

$$= \omega_0(100\times10^{-3}) - \frac{0.2\omega_0}{1 + 1\times10^{-4}\omega_0^2} = 0$$

$$\omega_0(100\times10^{-3}) = \frac{0.02\omega_0}{1 + 1\times10^{-4}\omega_0^2}$$

Cross multiply

$$\omega_0(100\times10^{-3})(1 + 1\times10^{-4}\omega_0) = 0.2\omega_0$$

$$= 0.1 + 1\times10^{-5}\omega_0^2 = 0.2$$

$$= 1\times10^{-5}\omega_0^2 = 0.2 - 0.1$$

$$\omega_0^2 = \frac{0.1}{1\times10^{-5}}$$

$$\omega_0^2 = 10000$$

$$\omega_0 = \sqrt{10000}$$

$$\omega_0 = 100\ \mathrm{rad/s}.$$