OJEIKERE OHIKHATEEME   17/ENG04/051   23rd April 2020

1) a) Given $R = 100k\Omega$ $C = 5nf$ $L = 20mH$

$\Rightarrow \omega_0 = 1/\sqrt{LC}$

$= 1/\sqrt{20\times10^{-3} \times 5\times10^{-9}}$

$= 100{,}000$ rad/s or 100krad/s

$\Rightarrow B = \frac{1}{RC} = \frac{1}{(100\times10^3)(5\times10^{-9})}$

$= \frac{1}{(100\times5\times10^{-6})}$

$= 2000$ rad/s or 2krad/s

$\Rightarrow Q = \frac{\omega_0}{B}$ from $B = \frac{\omega_0}{Q}$

$= \frac{100\times10^3}{2\times10^3} = 50$ [Q is dimensionless]

$\Rightarrow \omega_1$ & ω_2

$\omega_1 = \omega_0 - B/2$ Since $Q \geq 10$

$\omega_2 = \omega_0 + B/2$ Since $Q \geq 10$

$\therefore \omega_1 = (100\times10^3) - \left(\frac{2\times10^3}{2}\right) = 99$ krad/s

$\therefore \omega_2 = (100\times10^3) + \left(\frac{2\times10^3}{2}\right) = 101$ krad/s.

~~ii) Using the parallel addition form~~

ii) Using the parallel addition formula for admittance

$Y = \frac{1}{R} + j\omega C + \frac{1}{j\omega L}$

$\therefore Y = \frac{1}{10} + j\omega 0.1 + \frac{1}{2+j\omega 2}$

$\therefore Y = 0.1 + j\omega 0.1 + 1/2+j\omega 2$ Rationalizing

$Y = 0.1 + j\omega 0.1 + \left(\frac{1}{2+j\omega 2}\right) \times \frac{2-j\omega 2}{2-j\omega 2}$

$\therefore Y = 0.1 + j\omega 0.1 + \frac{2-j\omega 2}{4+4\omega^2}$

But at resonance, the magnitude of the imaginary part of admittance $= 0$

ie $Y = 0$ thus $(\omega = \omega_0)$

$\therefore 0 = W_0 \, 0.1 + \left(\frac{-W_0 2}{4 + W_0^2 4} \right)$

$\therefore 0 = W_0 \, 0.1 - \frac{W_0 2}{4 + W_0^2 4}$

$\therefore \frac{W_0 2}{4 + W_0^2 4} = 0.1 W_0$

$\therefore W_0 2 = (0.4 + 0.4 W_0^2) W_0$ divs by W_0

$\therefore 2 = 0.4 + 0.4 W_0^2$

$\therefore 2 - 0.4 = 0.4 W_0^2$

$1.6 = 0.4 W_0^2$

$1.6 / 0.4 = W_0^2$

$\therefore 4 = W_0^2$ thus $W_0 = \sqrt{4} = 2$ rad/s

$\therefore W_0 = 2$ rad/s.

iii) Using the series addition formular for impedance.

$Z = R + j\omega L + \frac{1}{j\omega C}$

$\therefore$ But R and C are in parallel thus we have

$Z = j\omega \, 100 \times 10^{-3} + \left(20 \times \frac{1}{j\omega (0.5 \times 10^{-3})} \div \left(\frac{20}{1} + \frac{1}{j\omega \, 0.5 \times 10^{-3}} \right) \right)$

5×10^{20}

Solving the brackets

$\Rightarrow \left(\frac{20}{j\omega (0.5 \times 10^{-3})} \div \frac{20(0.5 \times 10^{-3}) j\omega + 1}{j\omega (0.5 \times 10^{-3})} \right)$

$\Rightarrow \frac{20}{j\omega (0.5 \times 10^{-3})} \times \frac{(0.5 \times 10^{-3}) j\omega}{0.01 j\omega + 1}$

thus impedance equation becomes

5×10^{-10}

$Z = j\omega \, 100 \times 10^{-3} + \left(\frac{20}{\cancel{j\omega (0.5 \times 10^{-3})}} \times \frac{\cancel{(0.5 \times 10^{-3}) j\omega}}{1 + 0.01 j\omega} \right)$

$Z = j\omega \, 100 \times 10^{-3} + \frac{20}{1 + 0.01 j\omega}$

Rationalizing the values we get

$$Z = j\omega 100\times10^{-3} + \frac{20}{1+0.01j\omega} \times \frac{1-0.01j\omega}{1-0.01j\omega}$$

$$Z = j\omega 100\times10^{-3} + \frac{20 - 0.2j\omega}{1+1\times10^{-4}\omega^2}$$

But at resonance, the magnitude of the imaginary part of impedance = 0

ie $Z = 0$ thus $(\omega = \omega_0)$

$$0 = \omega_0 100\times10^{-3} - \frac{0.2\omega_0}{1+1\times10^{-4}\omega_0^2}$$

$$\therefore \frac{0.2\omega_0}{1+1\times10^{-4}\omega_0^2} = \omega_0 100\times10^{-3}$$

$$0.2\omega_0 = \omega_0(100\times10^{-3})(1+1\times10^{-4}\omega_0^2)$$

$$\therefore 0.2\omega_0 = 100\times10^{-3}\omega_0 + 100\times10^{-7}\omega_0^3$$ dbs by ω_0

$$0.2 = 100\times10^{-3} + 100\times10^{-7}\omega_0^2$$

$$0.2 - 100\times10^{-3} = 100\times10^{-7}\omega_0^2$$

$$1\times10^{-1} = 100\times10^{-7}\omega_0^2$$

$$\therefore \omega_0^2 = \frac{1\times10^{-1}}{100\times10^{-7}}$$

$$\omega_0^2 = 10{,}000$$ taking sqrt of bths.

$$\sqrt{\omega_0^2} = \sqrt{10{,}000}$$

$$\therefore \omega_0 = 100 \text{ rad/s}$$