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Assignment

Solution:

1 $R = 100k\Omega$, $L = 20mH$, $C = 5nf$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20\times10^{-3})(5\times10^{-9})}} = \frac{1}{\sqrt{1\times10^{-10}}} = 100000 rad/s$$

$\omega_0 = 100 krad/s$

$$Q = \frac{R}{\omega_0 L} = \frac{100\times10^3}{(100\times10^3)\times(20\times10^{-3})} = \frac{1}{20\times10^{-3}} = 50$$

$Q = 50$

$$B = \frac{\omega_0}{Q} = \frac{100\times10^3}{50} = 2000 = 2krad/s$$

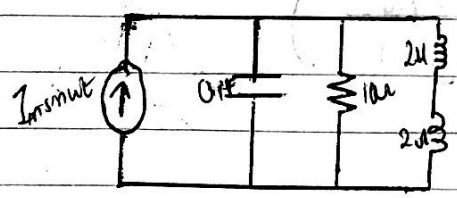
Since $Q > 10$, $\omega_1 = \omega_0 - \frac{B}{2} = 100\times10^3 - \frac{2000}{2} = 100\times10^3 - 1000 = 99000$

$\omega_1 = 99 krad/s$

$$\omega_2 = \omega_0 + \frac{B}{2} = 100\times10^3 + \frac{2000}{2} = 100\times10^3 + 1000 = 101000$$

$\omega_2 = 101 krad/s$.

2



$$Y = \frac{1}{10} + j\omega 0.1 + \frac{1}{2+j\omega 2} = 0.1 + j\omega 0.1 + \frac{1}{2+j\omega 2}$$

$$\frac{1}{2+j\omega 2} = \frac{1}{2+j\omega 2} \times \frac{2-j\omega 2}{2-j\omega 2} = \frac{2-j\omega 2}{4+4\omega^2}$$

$$Y = 0.1 + j\omega 0.1 + \frac{2-j\omega 2}{4+4\omega^2} = 0.1 + j\omega 0.1 + \frac{2}{4+4\omega^2} - \frac{j\omega 2}{4+4\omega^2}$$

$$Y = 0.1 + \frac{2}{4+4\omega^2} + j\omega 0.1 - \frac{j\omega 2}{4+4\omega^2} = 0.1 + \frac{2}{4+4\omega^2} + j\left(\omega 0.1 - \frac{\omega 2}{4+4\omega^2}\right)$$

At resonance, $Im(y) = 0$

Therefore $w_0 0.1 - \frac{w_0 2}{4+4w^2} = 0$

$0.1 - \frac{2}{4+4w^2} = 0$

$0.1 = \frac{2}{4+4w_0^2}$

$0.4 + 0.4w_0^2 = 2$

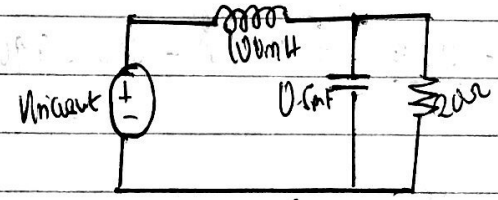
$0.4w_0^2 = 1.6$

$w_0^2 = 0.4 \times 10^1$

$= 4$

$w_0 = \sqrt{4}$

$= 2 \text{ rad/s}$



$$Z = jw100\times10^{-3} + \left(\frac{20}{jw(0.5\times10^{-3})} \div \left(20 + \frac{1}{0.5\times10^{-3}jw}\right)\right)$$

$$= jw(100\times10^{-3}) + \left(\frac{20}{jw(0.5\times10^{-3})} \div \left(\frac{0.01jw+1}{0.5\times10^{-3}jw}\right)\right)$$

$$= jw(100\times10^{-3}) + \left(\frac{20}{jw(0.5\times10^{-3})} \times \left(\frac{jw(0.5\times10^{-3})}{0.01jw+1}\right)\right)$$

$$= jw(100\times10^{-3}) + \left(\frac{20}{0.01jw+1}\right)$$

$$\frac{20}{jw0.01+1} = \frac{20}{0.01jw+1} \times \frac{0.01jw-1}{0.01jw-1}$$

$$= \frac{jw0.2-20}{-1\times10^{-4}w^2-1} = \frac{-20+jw0.2}{-1\times10^{-4}w^2-1}$$

$$Z = jw(100\times10^{-3}) + \frac{20}{1\times10^{-4}w^2+1} - \frac{0.2jw}{1+1\times10^{-4}w^2} = \frac{20}{1+1\times10^{-4}w^2} + jw(100\times10^{-3}) - \frac{0.2jw}{1+1\times10^{-4}w^2}$$

$$Z = \frac{20}{1+1\times10^{-4}\omega^2} + j\left(\omega(100\times10^{-3}) - \frac{0.2\omega}{1+1\times10^{-4}\omega^2}\right)$$

At resonance, the imaginary part of Z is 0

$$\therefore \omega(100\times10^{-3}) - \frac{0.2\omega}{1+1\times10^{-4}\omega^2} = 0$$

$$\omega(100\times10^{-3}) = \frac{0.2\omega_0}{1+1\times10^{-4}\omega_0^2}$$

$$100\times10^{-3}(1+1\times10^{-4}\omega_0^2) = 0.2$$

$$100\times10^{-3} + 1\times10^{-5}\omega^2 = 0.2$$

$$1\times10^{-5}\omega^2 = 0.2 - 100\times10^{-3} = 0.1$$

$$1\times10^{-5}\omega^2 = 0.1$$

$$\omega_0^2 = \frac{0.1}{10^{-5}}$$

$$\omega^2 = 10000$$

$$\omega = \sqrt{10000}$$

$$\omega = 100 \text{ rad/s}$$