

Assignment

1. A particle moves along a straight line with a velocity of $v = (4t - 3t^2)$ m/s where t is in seconds. Determine the position of the particle when $t = 4$ s. $s = 0$ when $t = 0$

Solution

$$v = (4t - 3t^2)$$

$$v = \frac{ds}{dt}$$

$$\int ds = \int v dt$$

$$s = \int v dt$$

$$s = \int_0^4 (4t - 3t^2) dt = \int_0^4 (4t - 3t^2) dt$$

$$s = \left(\frac{4t^2}{2} - \frac{3t^3}{3} \right)_0^4 = 2(4)^2 - 4^3 - 0 - 0 = -32 = 32$$

Position of particle = 32.

2. A particle moves along a straight line with a speed $v = (0.5t^3 - 8t)$ m/s where t is in seconds. Determine the acceleration of the particle when $t = 2$ s

Solution

$$v = (0.5t^3 - 8t)$$

$$a = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} = (1.5t^2 - 8) = (1.5t^2 - 8)$$

$$\text{when } t = 2$$

$$a = 1.5(2)^2 - 8 = -2 \text{ m/s}^2 = 2 \text{ m/s}^2$$

3. A particle moves along a straight line such that its acceleration is $a = (4t^2 - 2) \text{ m/s}^2$, where t is in seconds. When $t = 0$, the particle is located 2m to the left of the origin, and when $t = 2$, it is 20m to the left of the origin. Determine the position of the particle when $t = 4$ s.

Solution

$$a = (4t^2 - 2)$$

$$a = \frac{dv}{dt}$$

$$\int dv = \int a dt$$

$$v = \int (4t^2 - 2) dt$$

$$v = \frac{4t^3}{3} - 2t + C_1 \dots (1)$$

$$v = \frac{ds}{dt}$$

$$\int ds = \int v dt$$

$$s = \int \left(\frac{4t^3}{3} - 2t + C_1 \right) dt$$

$$s = \frac{t^4}{3} - t^2 + C_1 t + C_2 \dots (2)$$

$$\text{at } t = 0, s = -2$$

$$-2 = \frac{0^4}{3} - 0^2 + C_1(0) + C_2$$

$$C_2 = -2$$

$$\text{at } t = 2, s = -20$$

$$-20 = \frac{2^4}{3} - 2^2 + C_1(2) - 2$$

$$a = \frac{dv}{dt}$$

$$-20 = \frac{16}{3} - 4 + 2C_1 - 2$$

$$-20 = \frac{16}{3} - 6 + 2C_1$$

$$-20 - \frac{16}{3} + 6 = 2C_1$$

$$C_1 = -9.70$$

at $t = 4$ s

then

$$S = \frac{1}{3} (4)^3 - (4)^2 + (-2)(4) + (-9.7) = 28.7 \text{ m}$$

4.) A particle moves along a straight line with a velocity of $v = (20 - 0.05s^2) \text{ m/s}$, where s is in metres. Determine the acceleration of the particle at $s = 15 \text{ m}$.

Solution

$$v = \frac{ds}{dt}$$

$$dt = \frac{ds}{v}$$

$$a = \frac{dv}{dt}$$

$$dt = \frac{dv}{a}$$

$$\frac{ds}{v} = \frac{dv}{a}$$

$$a = v \frac{dv}{ds}$$

$$v = 20 - 0.05s^2$$

$$\frac{dv}{ds} = -2 \times 0.05s$$

$$ds$$

$$dv = -0.1s ds$$

$$a = \frac{v \times dv}{ds} \quad \dots (1)$$

Substitute $v = 20 - 0.05s^2$ and $dv = -0.1s ds$ in the eqn (1)

$$a = \frac{(20 - 0.05s^2) \times (-0.1s) ds}{ds}$$

$$a = -2s + 0.005s^3$$

$$\therefore \text{When } s = 15 \text{ m}$$

$$a = -2(15) + 0.005(15)^3$$

$$a = -13.125 \text{ m/s}^2$$

$$\therefore a = 13.125 \text{ m/s}^2$$