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Dept : ELECTRICAL/ELECTRONICS
Course : EEE 322 – Circuit Theory

(A) A parallel resonant circuit has $R = 100K\Omega$, $L = 20mH$ and $C = 5nF$. Calculate $\omega_0, \omega_1, \omega_2, Q$ and B.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(20\times10^{-3})(5\times10^{-9})}} = \frac{1}{\sqrt{1\times10^{-10}}} = \frac{1}{10^{-5}} = 100000\ rad/s$$

$$\therefore \omega_0 = 100\ Krad/s$$

$$Q = \frac{R}{\omega_0 L} = \frac{100\times10^{3}}{(100\times10^{3})\cdot(20\times10^{-3})} = \frac{1}{20\times10^{-3}} = 50$$

$$\therefore Q = 50$$

$$B = \frac{\omega_0}{Q} = \frac{100\times10^{3}}{50} = 2000 = 2\ Krad/s$$

Since $Q > 1.0$, $\omega_1 = \omega_0 - \frac{B}{2} = 100\times10^{3} = \frac{2000}{2}$

$$= 100\times10^{-3} - 1000 = 99000$$

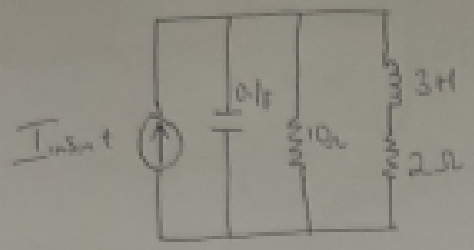
$$\omega_1 = 99\ Krad/s$$

$$\omega_2 = \omega_0 + \frac{B_0}{2} = 100\times10^{3} + \frac{2000}{2}$$

$$= 100\times10^{3} + 1000$$
$$= 101000$$

$$\omega_2 = 101\ Krad/s$$

(b)



$$Y = \frac{1}{10} + j\omega\, 0.1 + \frac{1}{2+j\omega 2} = 0.1 + j\omega 0.1 + \frac{1}{2+j\omega 2}$$

Rationalizing $\frac{1}{2+j\omega 2} = \frac{1}{2+j\omega 2} \times \frac{2-j\omega 2}{2-j\omega 2} = \frac{2-j\omega 2}{4-4j\omega+4j\omega+4\omega^2}$

$$= \frac{2-j\omega 2}{4+4\omega^2}$$

$$Y = 0.1 + j\omega 0.1 + \frac{2-j\omega 2}{4+4\omega^2} = 0.1 + j\omega 0.1 + \frac{2}{4+4\omega} = \frac{j\omega 2}{4+4\omega^2}$$

$$Y = \frac{0.1+2}{4+4\omega^2} + j\omega 0.1 = \frac{j\omega 2}{4+4\omega^2} = 0.1 + \frac{2}{4+4\omega^2} + j\left(\omega 0.1 - \frac{\omega^2}{4+4\omega^2}\right)$$

At resonance, $Im(Y) = 0$

Therefore, $\frac{\omega_0 0.1 - \omega_0^2}{4+4\omega_0^2} = 0$

$$\frac{0.1-2}{4+4\omega_0^2} = 0$$

$$0.1 = \frac{2}{4+4\omega_0^2}$$

$$0.4 + 0.4\omega_0^2 = 2$$

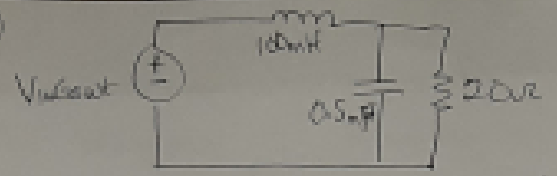
$$0.4\omega_0^2 = 1.6$$

$$\omega_0^2 = 4 \quad , \quad \sqrt{\omega^2} = \sqrt{4}$$

$$\omega_0 = 2$$

$$\omega_0 = 2 \text{ rad/s}$$

(c)



$$Z = j\omega 100\times10^{-3} + \left(\frac{20}{j\omega(0.5\times10^{-3})} \div \left(\frac{20}{1} + \frac{1}{0.5\times10^{-3}j\omega}\right)\right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \div \left(\frac{0.01j\omega}{0.5\times10^{-3}} + \frac{1}{j\omega}\right)\right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{j\omega(0.5\times10^{-3})} \times \frac{j\omega(0.5\times10^{-3})}{0.1j\omega+1}\right)$$

$$= j\omega(100\times10^{-3}) + \left(\frac{20}{0.01j\omega+1}\right)$$

Rationalising $\frac{20}{0.01j\omega+1} = \frac{20}{0.01j\omega+1} \times \frac{0.01j\omega-1}{0.01j\omega-1}$

$$= \frac{j\omega 0.2 - 20}{-1\times10^{-4}\omega^2 - j\omega 0.01 + j\omega 0.01 - 1}$$

$$= \frac{j\omega 0.2 - 20}{-1\times10^{-4}\omega^2 - 1} = \frac{-20 + j\omega 0.2}{-1\times10^{-4}\omega^2 - 1}$$

$$= -\frac{(20 - 0.2j\omega)}{-(1\times10^{-4}\omega^2+1)} = \frac{20 - 0.2j\omega}{1+1\times10^{-4}\omega^2} = \frac{20}{1+1\times10^{-4}\omega^2} + j\omega\frac{0.2}{1+1\times10^{-4}\omega^2}$$

$$Z = j\omega(100\times10^{-3}) + \frac{20}{1+1\times10^{-4}\omega^2} = \frac{0.2j\omega}{1+1\times10^{-4}\omega^2} = \frac{20}{1+1\times10^{-4}}$$

$$Z = \frac{20}{1+1\times10^{-4}\omega^2} + j\omega(100\times10^{-3}) - \frac{0.2j\omega}{1+1\times10^{-4}\omega^2}$$

$$Z = \frac{20}{1+1\times10} + j\left(\omega(100\times10^{-3}) - \frac{0.2\omega}{1+1\times10^{-4}\omega^2}\right)$$

At resonance the Imaginary part of Z, $\mathrm{Im}(z) = 0$

$$= \omega_0(1.50\times10^{-3}) - \frac{0.2\omega_0}{1+1\times10^{-9}\omega_0^2} = 0$$

$$\omega_0(1.50\times10^{-3}) = \frac{0.2\omega_0}{1+1\times10^{-9}\omega_0^2}$$

Cross multiply

$$\omega_0(1.50\times10^{-3})(1+1\times10^{-9}\omega_0) = 0.2\omega_0$$

$$= 0.1 + 1\times10^{-5}\omega_0^2 = 0.2$$

$$= 1\times10^{-5}\omega_0^2 = 0.2 - 0.1$$

$$\omega_0^2 = \frac{0.1}{1\times10^{-5}}$$

$$\omega_0^2 = 10000$$

$$\omega_0 = \sqrt{10000}$$

$$\omega_0 = 100\ \mathrm{rad/s}$$