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Computer Engineering
18/ENG02/104
EEE 322

Q1)

a) $R = 100k\Omega$

$L = 20mH$

$C = 5\mu F$

Calculate $\omega_0, \omega_1, \omega_2$, Q and B

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 5 \times 10^{-9}}}$$

$= 100k\,rad/s$

$$\omega_1 = \frac{-1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}} + \sqrt{\left(\frac{1}{2 \times 100 \times 10^3 \times 5 \times 10^{-9}}\right)^2 + \frac{1}{20 \times 10^{-3} \times 5 \times 10^{-9}}}$$

$= 100005$

$$\omega_1 = \frac{-1}{2RC} + 100005$$

$$= \frac{-1}{2 \times 100 \times 10^3 \times 5 \times 10^{-9}} + 100005$$

$= -1000 + 100005$

$= 99005$

$= 99k\,rad/s$

$$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$1000 + 100005$

101000

$101k\,rad/s$

$$Q = \frac{\omega_0}{B}$$

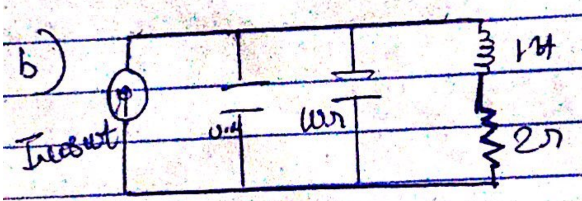
$$B = \omega_2 - \omega_1$$
$$= (101 - 99)\ krad/s$$
$$= 2\ krad/s$$

$$Q = \frac{100 \times 10^3}{2 \times 10^3}$$

$$= 50$$

$\omega_0 = 100\ krad/s$
$\omega_1 = 99\ krad/s$
$\omega_2 = 101\ krad/s$
$Q = 50$
$B = 2\ krad/s$

b)



$$Y = \frac{1}{\omega} + j\omega 0.1 + \frac{1}{2 + j\omega 2}$$

(Rationalize)

$$\frac{1}{2 + j\omega^2} = \frac{1}{2 + j\omega^2} \times \frac{2 - j\omega^2}{2 - j\omega^2}$$
$$= \frac{2 - j\omega 2}{4 + 4\omega^2}$$

$$Y = \frac{1}{\omega} + j\omega 0.1 + \frac{2 - j\omega 2}{4 + 4\omega^2}$$

$$= \frac{1}{\omega} + j\omega 0.1 + \frac{2}{4 + 4\omega^2} - \frac{2j\omega}{4 + 4\omega^2}$$

$$= \frac{1}{\omega} + \frac{2}{4 + 4\omega^2} + j\omega 0.1 - \frac{2j\omega}{4 + 4\omega^2}$$

$$= \frac{1}{\omega} + \frac{2}{4 + 4\omega^2} + j\left[\omega 0.1 - \frac{2\omega}{4 + 4\omega^2}\right]$$

At resonance, the imaginary part of $Z_{in}(4) = 0$

$$\omega_1 0.1 - \frac{2\omega}{4+4\omega_1^2} = 0$$

$$\frac{\omega_1}{\omega_1}\left[0.1 - \frac{2}{4+4\omega_1^2}\right] = \frac{0}{\omega_1}$$

$$0.1 - \frac{2}{4+4\omega_1^2} = 0$$

$$0.4 + 0.4\omega_1^2 = 2$$

$$\frac{0.4\omega_1^2}{0.4} = \frac{2-0.4}{0.4}$$

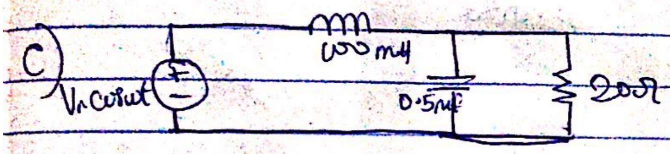
$$\omega_1^2 = \frac{1.6}{0.4}$$

$$\omega_1^2 = 4$$

$$\omega_0 = \sqrt{4}$$

$$\omega_1 = 2\,rad/s$$

c)



$$= j\omega(100\times10^{-3}) \times \left[\frac{20}{j\omega 0.5\times10^{-3}} \div \left(\frac{20}{1} + \frac{1}{0.5\times10^{-3} j\omega}\right)\right]$$

$$= j\omega(100\times10^{-3}) + \frac{20}{j\omega 0.5\times10^{-3}} \div \frac{0.01j\omega+1}{0.5\times10^{-3} j\omega}$$

$$= j\omega(100\times10^{-3}) + \left[\frac{20}{j\omega(0.5\times10^{-3})} \times \frac{0.5\times10^{-3} j\omega}{0.01j\omega+1}\right]$$

$$= j\omega\, 100\times10^{-3} + \frac{20}{0.01j\omega+1}$$

$$\frac{20}{0.01j\omega+1} \times \frac{0.01j\omega-1}{0.01j\omega-1} \quad \text{(Rationalize)}$$

$$\frac{0.2j\omega - 20}{1\times10^{-4}\omega^2+1} = \frac{-0.2j\omega}{1\times10^{-4}\omega^2+1} + \frac{20}{1\times10^{-4}\omega^2+1}$$

$$= j\omega\, 100\times10^{-3} + \frac{20}{1\times10^{-4}+1} + 1\left(\frac{-0.2\omega j}{1\times10^{-4}\omega^2+1}\right)$$

$$= \frac{+20}{1\times10^{-4}+1} + \left[\frac{-0.2\omega j}{1\times10^{-4}\omega^2+1}\right] + j\omega\, 100\times10^{-3}$$

$= \frac{+20}{1\times10^{-4}+1} + j\left[\frac{-0.2\omega}{1\times10^{-4}\omega^2+1} + \omega 100\times10^{-3}\right]$

At resonance (imaginary part (z) = 0

$\frac{-0.2\omega_0}{1\times10^{-4}\omega_0^2+1} + \omega_0 100\times10^{-3} = 0.$

$\omega_0 100\times10^{-3} - \frac{0.2\omega_0}{1\times10^{-4}\omega_0^2+1} = 0$

Divide by ω_0

$\frac{\omega_0}{\omega_0} 100\times10^{-3} - \frac{0.2\omega_0}{1\times10^{-4}\omega_0^2+1} \times \frac{1}{\omega_0} = 0$

$100\times10^{-3} = \frac{0.2}{1\times10^{-4}\omega_0^2+1}$

$1\times10^{-5}\omega_0^2 + 100\times10^{-3} = 0.2$

$\omega_0^2 = \frac{0.2 - 100\times10^{-3}}{1\times10^{-5}}$

$\omega_0^2 = \frac{0.1}{1\times10^{-6}} = 10,000$

$\omega_0 = \sqrt{10,000}$

$= 100 rad/s$