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1. If $A = 3i + 7j - 2k$, $B = i + 3j + k$, $C = 9i - 4j + 6k$, find the angle between (i) A and C ii. B and C iii. unit vector in the direction of (A+B+C).

solution.

i. $A \times C$

Angle = $\sin\theta$

$$\sin\theta = \frac{|\bar{A} \times \bar{C}|}{|A||C|}$$

$$|\bar{A} \times \bar{C}| = \sqrt{\quad}$$

$$\bar{A} \times \bar{C} = \begin{vmatrix} i & j & k \\ 3 & 7 & -2 \\ 9 & -4 & 6 \end{vmatrix}$$

$$= i\begin{vmatrix} 7 & -2 \\ -4 & 6 \end{vmatrix} - j\begin{vmatrix} 3 & -2 \\ 9 & 6 \end{vmatrix} + k\begin{vmatrix} 3 & 7 \\ 9 & -4 \end{vmatrix} = i(42-8) - j(18+18) + k(-12-63)$$

$$A \times C = 34i - 36j - 75k \therefore |A \times C| = \sqrt{(34)^2 + (-36)^2 + (-75)^2} = 89.87$$

$$|A| = \sqrt{(3)^2 + (7)^2 + (-2)^2} = \sqrt{9 + 49 + 4} = \sqrt{62}$$

$$|A| = \sqrt{62}$$

$$|C| = \sqrt{(9)^2 + (-4)^2 + (6)^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$$

$$|C| = \sqrt{133}$$

$$\sin\theta = \frac{|\bar{A} \times \bar{C}|}{|A||C|} = \frac{89.87}{\sqrt{62} \times \sqrt{133}}$$

$$= \frac{89.87}{90.81} = 0.9897$$

$$\sin\theta = 0.9897$$

$$\theta = \sin^{-1} 0.9897$$

$$\theta = 81.77^\circ$$

ii. B and C

Angle = $\sin\theta$

$\sin\theta = \frac{|B \times C|}{|B|\,|C|}$

$$B \times C = \begin{vmatrix} i & j & k \\ 1 & 3 & 7 \\ 9 & -4 & 6 \end{vmatrix}$$

$$= i\begin{vmatrix} 3 & 7 \\ -4 & 6 \end{vmatrix} - j\begin{vmatrix} 1 & 7 \\ 9 & 6 \end{vmatrix} + k\begin{vmatrix} 1 & 3 \\ 9 & -4 \end{vmatrix} = i(18+28) - j(6-63) + k(-4-27)$$

$B \times C = 46i + 57j - 31k$

$|B \times C| = \sqrt{(46)^2 + (57)^2 + (-31)^2}$

$= \sqrt{2116 + 3249 + 961}$

$= \sqrt{6326}$

$|B \times C| = 79.54$

$|B| = \sqrt{(1)^2 + (3)^2 + (7)^2}$ $\quad |C| = \sqrt{133}$

$= \sqrt{1+9+49}$

$|B| = \sqrt{59}$

$\sin\theta = \frac{|B \times C|}{|B|\,|C|}$

$\sin\theta = \frac{79.54}{\sqrt{59} \times \sqrt{133}} = \frac{79.54}{88.58} = 0.8980$

$\sin\theta = 0.8980$

$\theta = \sin^{-1} 0.8980$

$\theta = 63.90^\circ$

iii. $(A+B+C) = (3i+7j-2k) + (i+3j+7k) + (9i-4j+6k)$

$= 13i + 6j + 11k$

$(A+B+C) = 13i + 6j + 11k$

$|(A+B+C)| = \sqrt{(13)^2 + (6)^2 + (11)^2} = \sqrt{169 + 36 + 121} = \sqrt{326}$

$|A+B+C| = \sqrt{326}$

Unit vector $(\hat{U}) = \frac{13i + 6j + 11k}{\sqrt{326}}$

$\hat{U} = \frac{13i}{\sqrt{326}} + \frac{6j}{\sqrt{326}} + \frac{11k}{\sqrt{326}}$

3. If $A = 4i+2j-4k$, $B = 8i-9j+k$, $C = i+4j-3k$, find triple product $(A \times B) \times C$

Solution.

$$A \times B = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -9 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & -4 \\ -9 & 1 \end{vmatrix} - j\begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k\begin{vmatrix} 4 & 2 \\ 8 & -9 \end{vmatrix}$$

$$= i(2-36) - j(4+32) + k(-36-16)$$

$$A \times B = -34i - 36j - 52k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -34 & -36 & -52 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= i\begin{vmatrix} -36 & -52 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} -34 & -52 \\ 1 & -3 \end{vmatrix} + k\begin{vmatrix} -34 & -36 \\ 1 & 4 \end{vmatrix}$$

$$= i(108+208) - j(102+52) + k(-136+36)$$

$$(A \times B) \times C = 316i - 154j - 100k$$

2. A particle moves along a curve, $X = -8t^2$, $y = t^2 - 4t$, $Z = t+1$, where t is time. Find modulus of acceleration at $t = 1$

Solution.

$$X = -8t^2 \quad y = t^2 - 4t \quad Z = t+1$$

$$\frac{dA}{dt} = -16ti + 2t - 4j + 1k$$

$$\frac{d^2A}{dt^2} = -16i + 2j + k$$

$$\left|\frac{d^2A}{dt^2}\right| \text{ at } t=1 = \sqrt{(-16)^2 + (2)^2 + (1)^2}$$

$$= \sqrt{256+4+1}$$

$$= \sqrt{261}$$

$$\therefore \left|\frac{d^2A}{dt^2}\right| = 3\sqrt{29}$$