

NKOKORIE PASCAL CHINAMDI

Mechatronics Engineering

19/ENG051043

1) $A = 3i + 7j - 2k$, $B = 1 + 3j + k$, $C = 9i - 4j + 6k$.

Soln

i) $A \times C$.

Angle = $\sin \theta$.

$$\sin \theta = \frac{|A \times C|}{|A| |C|}$$

$$A \times C = \begin{vmatrix} i & j & k \\ 3 & 7 & -2 \\ 9 & -4 & 6 \end{vmatrix}$$

$$= i \begin{vmatrix} 7 & -2 \\ -4 & 6 \end{vmatrix} - j \begin{vmatrix} 3 & -2 \\ 9 & 6 \end{vmatrix} + k \begin{vmatrix} 3 & 7 \\ 9 & -4 \end{vmatrix}$$

$$= i(42 - 8) - j(18 + 18) + k(-12 - 63)$$

$$A \times C = 34i - 36j - 75k$$

$$|A \times C| = \sqrt{(34)^2 + (-36)^2 + (-75)^2}$$

$$|A \times C| = 89.87$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{9 + 49 + 4} = \sqrt{62}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$$

$$\sin \theta = \frac{|A \times C|}{|A| |C|} = \frac{89.87}{\sqrt{62} \times \sqrt{133}} = \frac{89.87}{90.81} = 0.9897$$

$$\sin \theta = 0.9897$$

$$\theta = \sin^{-1}(0.9897)$$

$$\theta = \underline{\underline{81.77^\circ}}$$

ii) $B \times C$

$$B \times C = \begin{vmatrix} i & j & k \\ 1 & 3 & 7 \\ 9 & -4 & 6 \end{vmatrix}$$

$$= i \begin{vmatrix} 3 & 7 \\ -4 & 6 \end{vmatrix} - j \begin{vmatrix} 1 & 7 \\ 9 & 6 \end{vmatrix} + k \begin{vmatrix} 1 & 3 \\ 9 & -4 \end{vmatrix}$$

$$= i(18 + 28) - j(6 - 63) + k(-4 - 27)$$

$$B \times C = 46i + 57j - 31k$$

$$|B \times C| = \sqrt{46^2 + 57^2 + (-31)^2} = \sqrt{2116 + 3249 + 961}$$

$$= \sqrt{6362} = 79.54$$

$$\text{Angle} = \sin \theta$$

$$\sin \theta = \frac{|B \times C|}{|B| |C|}$$

$$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{1 + 9 + 49} = \sqrt{59}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$$

$$\sin \theta = \frac{|B \times C|}{|B| |C|} = \frac{79.54}{\sqrt{59} \times \sqrt{133}} = \frac{79.54}{88.58} = 0.8980$$

$$\sin \theta = 0.8980$$

$$\theta = \sin^{-1}(0.8980)$$

$$\theta = \underline{\underline{63.9^\circ}}$$

$$14) (A+B+C) = (3i + 7j - 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$$

$$= 13i + 6j + 11k$$

$$|(A+B+C)| = \sqrt{13^2 + 6^2 + 11^2} = \sqrt{169 + 36 + 121} = \sqrt{326}$$

$$|(A+B+C)| = \sqrt{326}$$

$$\text{Unit Vector } (\hat{u}) = \frac{13i + 6j + 11k}{\sqrt{326}}$$

$$\hat{u} = \frac{13i}{\sqrt{326}} + \frac{6j}{\sqrt{326}} + \frac{11k}{\sqrt{326}}$$

$$2) x = -8t^2 - 4 = t^2 - 4t, \quad -2 = t + 1$$

$$\frac{dx}{dt} = -16t + (2t - 4)j + 1k$$

$$\frac{d^2x}{dt^2} = -16i + 2j + k$$

$$\left| \frac{d^2x}{dt^2} \right| = \sqrt{(-16)^2 + (2)^2 + 1^2} = \sqrt{256 + 4 + 1}$$

$$\left| \frac{d^2x}{dt^2} \right| \text{ at } t=1 = \sqrt{261}$$

$$= \underline{\underline{3\sqrt{29}}}$$

$$3) A = 4i + 2j - 4k, B = 8i - 9j + k, C = i + 4j - 3k$$

Solu

$$(A \times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -9 & 1 \end{vmatrix}$$

$$= i \begin{vmatrix} 2 & -4 \\ -9 & 1 \end{vmatrix} - j \begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k \begin{vmatrix} 4 & 2 \\ 8 & -9 \end{vmatrix}$$

$$= i(2 - 36) - j(4 + 32) + k(-36 - 16)$$

$$(A \times B) = -34i - 36j - 52k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -34 & -36 & -52 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= i \begin{vmatrix} -36 & -52 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -34 & -52 \\ 1 & -3 \end{vmatrix} + k \begin{vmatrix} -34 & -36 \\ 1 & 4 \end{vmatrix}$$

$$= i(108 + 208) - j(102 + 52) + k(-136 + 36)$$

$$(A \times B) \times C = \underline{\underline{316i - 154j - 100k}}$$