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MAT 102

19/ENG05/052

MELHATRONICS ENGINEERING

1) If $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$, find the angle between (i) A and C (ii) B and C (iii) the unit vector in the direction of $(A+B+C)$

solution

$$\cos \theta = \frac{A \cdot C}{|A||C|}$$

$$\cos \theta = \frac{27 - 28 - 12}{\sqrt{9 + 49 + 4} \cdot \sqrt{81 + 16 + 36}}$$

$$\cos \theta = \frac{-13}{\sqrt{62} \cdot \sqrt{133}}$$

$$\cos \theta = \frac{-13}{90.81}$$

$$\theta = \cos^{-1} - 0.1432$$

$$\theta = 98.2^\circ$$

$$\textcircled{\text{ii}} \cos \theta = \frac{B \cdot C}{|B||C|}$$

$$\cos \theta = \frac{9 - 12 + 42}{\sqrt{1+9+49} \cdot \sqrt{81+16+36}}$$

$$\cos \theta = \frac{39}{\sqrt{59} \cdot \sqrt{133}}$$

$$\cos \theta = \frac{39}{88.58}$$

$$\cos \theta = 0.4403$$

$$\theta = \cos^{-1} 0.4403$$

$$\theta = 63.88^\circ$$

$$\textcircled{\text{iii}} \rho_{(A+B+C)} \\ A+B+C = (3+1+4)i + (7+3-4)j + (-2-4+7)k \\ 13i + 6j + k$$

$$\rho_{(A+B+C)} = \frac{(\bar{A} + \bar{B} + \bar{C})}{|A+B+C|}$$

$$\rho_{(A+B+C)} = \frac{13i + 6j + k}{\sqrt{169 + 36 + 1}} = \frac{13i + 6j + k}{\sqrt{206}}$$

2) A particle moves along a curve, $x = -8t^2$, $y = t^2$, $z = t+1$, where t is time. Find the modulus of acceleration at $t=1$

$$r = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$r = -8t^2\mathbf{i} + t^2\mathbf{j} + (t+1)\mathbf{k}$$

$$\frac{dr}{dt} = -16t\mathbf{i} + 2t\mathbf{j} + \mathbf{k}$$

$$\left. \frac{d^2r}{dt^2} \right|_{t=1} = -16\mathbf{i} + 2\mathbf{j}$$

$$= \sqrt{256 + 4}$$

$$= \sqrt{260}$$

$$= 2\sqrt{65}$$

3) If $A = 4\mathbf{i} + 9\mathbf{j} + 4\mathbf{k}$, $B = 8\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $C = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$. Find the vector triple product $(A \times B) \times C$

$$A \times B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 9 & 4 \\ 8 & 2 & 1 \end{vmatrix}$$

$$= \mathbf{i} \begin{vmatrix} 9 & 4 \\ 2 & 1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 4 & 4 \\ 8 & 1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 4 & 9 \\ 8 & 2 \end{vmatrix}$$

$$i(2-8) - j(4+32) + k(-8+16)$$

$$= -6i - 36j + 8k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & 8 \\ 1 & 4 & -3 \end{vmatrix}$$

$$i \begin{vmatrix} -36 & 8 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -6 & 8 \\ 1 & -3 \end{vmatrix} + k \begin{vmatrix} -6 & -36 \\ 1 & 4 \end{vmatrix}$$

$$i(108 - 32) - j(18 - 8) + k(-24 + 36)$$

$$= 72i - 10j + 12k$$