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Question

Integrate the following with respect to their Variables.

1 $x^{1/2} \ln x$

2 $2\cos 6t \cos t$

3 $\sin^3 x \cos^4 x$

Solutions

1 $\int x^{1/2} \ln x \, dx$

$u = \ln x \qquad dv = x^{1/2}$

$du/dx = 1/x \qquad v = \frac{2x^{3/2}}{3}$

$du = dx/x$

$\int u dv = uv - \int v du$

$= \ln x \cdot \frac{2x^{3/2}}{3} - \int \frac{2x^{3/2}}{3} \cdot \frac{dx}{x}$

$= \frac{2x^{3/2} \ln x}{3} - \int \frac{2x^{1/2}}{3} dx$

$= \frac{2x^{3/2} \ln x}{3} - \left[\frac{2x^{3/2}}{3 \times 3/2} + c \right]$

$$\boxed{\int x^{1/2} \ln x \, dx = \frac{2x^{3/2} \ln x}{3} - \frac{4x^{3/2}}{9} + c}$$

2 $\int 2\cos 6t \cos t \, dt$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

Where $A = 6t$; $B = t$

$\therefore \cos 6t \cos B = \frac{1}{2}[\cos 7t + \cos 5t$

$\int 2 \cdot \frac{1}{2}[\cos 7t + \cos 5t] \, dt$

$= \int \cos 7t + \cos 5t \, dt$

$= \int \cos 7t \, dt + \int \cos 5t \, dt$

$= \left[\frac{\sin 7t}{7} + \frac{\sin 5t}{5}\right] + C$

$\therefore \boxed{\int 2\cos 6t \cos t \, dt = \frac{\sin 7t}{7} + \frac{\sin 5t}{5} + C}$

6

3 $\int \sin^3 x \cos^4 x \, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x \quad \Rightarrow du = -\sin x \, dx \Rightarrow dx = \frac{du}{-\sin x}$

$\sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 x = 1 - \cos^2 x$

$= \int \sin x \sin^2 x \, u^4 \cdot \frac{du}{-\sin x}$

$= \int u^4 \cdot -\sin^2 x \, du$

$= -\int u^4 \sin^2 x \, du$

$= -\int (1 - \cos^2 x) u^4 \, du$

$= -\int (1 - u^2) u^4 \, du$

$= \int (u^6 - u^4) \, du$

$= \int u^6 du - \int u^4 \, du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C$

$\therefore \boxed{\int \sin^3 x \cos^4 x \, dx = \frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + C}$