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S/N: 36 MAT 104 Mechanical Engr

Integrate the following with respect to their variables.

1. $x^{1/2} \ln x$

$u = \ln x$ $\quad dv = x^{1/2}$

$du = \frac{1}{x}$ $\quad v = \frac{2x\sqrt{x}}{3}$

$\int u\,dv = uv - \int v\,du$

$= \frac{2x\sqrt{x}\ln x}{3} - \int \frac{2x\sqrt{x}}{3} \times \frac{1}{x}$

$= \frac{2x\sqrt{x}\ln x}{3} - \int \frac{2x^{1/2}}{3}$

$= \frac{2x\sqrt{x}\ln x}{3} - \frac{2}{3} \times \int 2x^{1} x^{1/2}$

$= \frac{2x\sqrt{x}\ln x}{3} - \frac{2}{3} \times \frac{x\sqrt{x}}{3}$

$= \frac{2x\sqrt{x}\ln x}{3} - \frac{4x\sqrt{x}}{9} + C$

2 $2\cos 6t \cos t$

$A = 6t \quad B = t$

$\frac{1}{2}(\cos(A-B) + \cos(A+B))$

$2 \times \frac{1}{2}\int(\cos 5t + \cos 7t)$

$\int \cos 5t + \cos 7t$

$\frac{\sin 5t}{5} + \frac{\sin 7t}{7} + C$

③ $\int \sin^3 x \cos^4 x$

Recall $\cos^2 x + \sin^2 x = 1$

$1 - \cos^2 x = \sin^2 x$

$\sin^3(x) \cos^4(x) =$

$\sin(x) \cdot \sin^2(x) \cdot \cos^4(x)$

$= \sin(x)[1 - \cos^2(x)]\cos^4(x)$

Let $u = \cos(x)$

$\frac{du}{dx} = -\sin(x) \Rightarrow -du = \sin(x)dx$

$$= \int \sin(x)\left[1-u^2\right]u^4$$

$$= \int \sin(x)\left[1-u^2\right]u^4\,dx$$

$$= \int (1-u^2)u^4 \sin(x)\,dx$$

$$= \int (1-u^2)u^4 \cdot (-du)$$

$$\int (u^6 - u^4)\,du$$

$$= \frac{u^{6+1}}{6+1} - \frac{u^{4+1}}{4+1}$$

$$= \frac{u^7}{7} - \frac{u^5}{5}$$

$$= \frac{\cos^7 x}{7} - \frac{\cos}{5} + C$$