Name: Udoh Timah Victoria

Dept: Petroleum Engineering

Matric NO: 19/ENG071022

Lecturer's Name: Mr okuntola Joseph Temitayo

Date of submission: 30-04-2020

Questions

1) If $A = 3i + 7j - 2K$, $B = i + 3j + 7K$, $C = 9i - 4j + 6K$, find the angle between. (i) A and C (ii) B and C (iii) The unit vector in the direction of $(A + B + C)$.

Solution

(i) Angle Between A and C

$A = 3i + 7j - 2K$ and $C = 9i - 4j + 6K$

$\overline{A} \cdot \overline{C} = (3i + 7j - 2K) \cdot (9i - 4j + 6K)$

$= 27i + 28j - 12K$

$\overline{A} \cdot \overline{C} = -13$

The angle $\cos\theta$

$|A| = \sqrt{3^2 + 7^2 + 2^2} = \sqrt{62}$

$|C| = \sqrt{9^2 + 4^2 + 6^2} = \sqrt{133}$

$\cos\theta = \dfrac{\overline{A} \cdot \overline{C}}{|A||C|} = \dfrac{-13}{62 \times 133} = \dfrac{-13}{90.81} = -0.143$

$\cos\theta = -0.143$

$\theta = \cos^{-1} -0.143$

$\theta = 98.2°$

ii) B and C $B = i + 3j + 7K$ and $C = 9i - 4j + 6K$

$\overline{B} \cdot \overline{C} = (i + 3j + 7K) \cdot (9i - 4j + 6K)$

$= 9i - 12j + 42K = 39$

The angle $\cos\theta$

$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$

$|C| = \sqrt{9^2 + 4^2 + 6^2} = \sqrt{133}$

$\cos\theta = \dfrac{\overline{B} \cdot \overline{C}}{|B||C|} = \dfrac{39}{59 \times 133} = \dfrac{39}{88.58} = 0.440$

$\cos\theta = 0.440$, $\theta = \cos^{-1} 0.440$ $\theta = 63.9°$

iii) The unit vector in the direction of $(A+B+C)$.

$(A+B+C)$

$(3i + 7j - 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$

$13i + 6j + 11k$

$(A+B+C) = 13i + 6j + 11k$

$= \sqrt{13^2 + 6^2 + 11^2} = 18.06$

$e_{A+B+C} = \dfrac{13i + 6j + 11k}{18.06}$ OR $e_{A+B+C} = \dfrac{13}{18.06}i + \dfrac{6}{18.06}j + \dfrac{11}{18.06}k$

2) A particle moves along a curve, $X = -8t^2$, $y = t^2 - 4t$, $z = t+1$, where t is time. find the modulus of acceleration at $t=1$.

Solution

$x = -8t^2$, $y = t^2 - 4t$, $z = t+1$

$r = (-8t^2)i + (t^2 - 4t)j + (t+1)k$

$\dfrac{dr}{dt} = 16ti + (2t-4)j + k$

$\dfrac{d^2r}{dt^2} = 16i + 2j + k$ //

$\left|\dfrac{d^2r}{dt^2}\right|_{t=1} = \sqrt{16^2 + 2^2 + 1^2} = 16.2$ //

3) If $A = 4i + 2j - 4k$, $B = 8i - 9j + k$, $C = i + 4j - 3k$, find the Vector triple product $(A\times B)\times C$

Solution

$A = 4i + 2j - 4k$, $B = 8i - 9j + k$, $C = i + 4j - 3k$

$(A\times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -1 & 9 \end{vmatrix} = i(2+4) - j(4+32) + k(-4-16)$
$= 6i - 36j - 20k$ //

$(A\times B)\times C = \begin{vmatrix} i & j & k \\ 6 & -36 & -20 \\ 1 & 4 & -3 \end{vmatrix} = i(108-80) - j(-18+20) + k(24+36)$
$= 28i - 2j + 60k$ //