

$$1. \quad \lim_{x \rightarrow 0} \frac{4x^2 - \sin(x)}{x^2} = \lim_{x \rightarrow 0} \left(\frac{4x^2}{x^2} - \frac{\sin(x)}{x^2} \right)$$

$$= \lim_{x \rightarrow 0} 4 - \lim_{x \rightarrow 0} \frac{\sin(x)}{x^2} = 4 - \lim_{x \rightarrow 0} \frac{\sin(x)}{x^2}$$

$$2. \quad y = 2x^2 \cos(2x)$$

$$\frac{dy}{dx} = \frac{d}{dx} (2x^2 \cos(2x)) = 2x \frac{d}{dx} (\cos(2x)) + \cos(2x) \frac{d}{dx} (2x^2)$$

$$= 2x \left(-\sin(2x) \cdot 2 \right) + \cos(2x) \cdot 4x$$

$$= -4x \sin(2x) + 4x \cos(2x)$$

$$\frac{dy}{dx} = 4x (\cos(2x) - \sin(2x))$$

$$y = \cos(3x^2 + 6x)$$

$$\frac{d}{dx} \cos(3x^2 + 6x)$$

(Chain rule)

$$\frac{d}{dx} (f(g)) = \frac{d}{dy} (f(y)) \times \frac{dy}{dx} (g)$$

$$y = 3x^2 + 6x$$
$$\frac{d}{dx} (\cos(y)) \times \frac{d}{dx} (3x^2 + 6x)$$

$$= -\sin(y) \times \frac{d}{dx} (3x^2 + 6x)$$

$$= -\sin(y) \times (6x + 6)$$

$$= -\sin(3x^2 + 6x) \times (6x + 6)$$

$$= -6 \sin(3x^2 + 6x) (x + 1)$$

$$\int (4x + 1) dx = 3 dx$$

$$3 \times \int (4x + 1) dx =$$

$$3 \left(\int 4x dx + \int 1 dx \right)$$

$$= 3 \left(2x^2 + x \right)$$

$$= \frac{6x^3 + 3x}{x^2 + 3x + 2} + C$$

$$\begin{aligned} \text{Q. } \int e^{5x} + 7x^4 - 5 \sin 2x + \cos 8x \, dx \\ = \int e^{5x} dx + \int 7x^4 dx - \int 5 \sin 2x dx + \int \cos 8x dx \\ \frac{e^{5x}}{5} + \frac{7x^5}{5} + \frac{\cos(2x)}{2} + \frac{\sin(8x)}{8} \\ \frac{e^{5x}}{5} + \frac{7x^5}{5} + \frac{\cos 2x}{2} + \frac{\sin 8x}{8} + C \end{aligned}$$

$$\begin{aligned} \int \frac{\sqrt{9+x^2}}{2} dx &= \int \frac{1}{2} \sqrt{9+x^2} dx \\ &= \frac{1}{2} \int \sqrt{9+x^2} dx \end{aligned}$$

$$\begin{aligned} &= \frac{1}{2} \times \int \sqrt{t} dt \\ &= \frac{1}{2} \times \frac{2}{3} t^{3/2} \end{aligned}$$

$$\begin{aligned} &= \frac{1}{3} \sqrt{9+x^2} \\ \text{replace } & \frac{1}{3} \sqrt{9+x^2} \text{ for } \sqrt{9+x^2} \\ &= \frac{1}{3} \sqrt{9+x^2} \end{aligned}$$

