

MAT 102 30th April
 AMULUNKO CHUKHUEBUKA OLIVER
 MECHATRONICS ENGINEERING

19/ENG051014

1. $\vec{A} = 3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$, $\vec{B} = \mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$
 $\vec{C} = 9\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ find the angle between

i) A and C

$$A \cdot C = |A| |C| \cos \theta$$

$$A \cdot C = (3\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}) \cdot (9\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) = 27 - 28 - 12$$

$$A \cdot C = -13$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{62}$$

$$|B| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\theta = \cos^{-1} \frac{A \cdot C}{|A| |C|}$$

$$\theta = \cos^{-1} \frac{-13}{\sqrt{62} \times \sqrt{133}}$$

$$\theta = \cos^{-1} -0.14315$$

$$\theta = 98.2307^\circ$$

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ii) B and C

$$B \cdot C = |B| |C| \cos \theta$$

$$B \cdot C = (\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}) \cdot (9\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}) = 9 - 12 + 42 = 39$$

$$B \cdot C = 39$$

$$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\theta = \cos^{-1} \frac{B \cdot C}{|B| |C|}$$

$$\theta = \cos^{-1} \frac{39}{\sqrt{59} \times \sqrt{133}}$$

$$\theta = \cos^{-1} 0.44026$$

$$\theta = 63.879^\circ$$

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The unit vector in the direction of $(A+B+C)$

$$e_{(A+B+C)} = \frac{(\bar{A} + \bar{B} + \bar{C})}{|\bar{A} + \bar{B} + \bar{C}|}$$

$$A+B+C = (3i + 7j - 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$$

$$A+B+C = 3i + 6j + 11k$$

$$|A+B+C| = \sqrt{3^2 + 6^2 + 11^2} = \sqrt{326}$$

$$e_{(A+B+C)} = \frac{3i + 6j + 11k}{\sqrt{326}} = \frac{3i + 6j + 11k}{18.0555}$$

2 $r = -8t^2 i + (t^2 - 4t) j + (t+1) k$

$$v = \frac{dr}{dt} = -16t i + (2t - 4) j + 1k$$

$$a = \frac{dv}{dt} = -16i + 2j + 0k$$

at $t=1$

$$a = \sqrt{(-16)^2 + (2)^2} = \underline{\underline{16.1245 \text{ ms}^{-2}}}$$

3 If $A = 4i + 2j - 4k$, $B = 8i - 2j + k$, $C = i + 4j - 3k$
find the vector triple product $(A \times B) \times C$

$$(A \times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$i \begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - j \begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k \begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$i(2 - 8) - j(4 + 32) + k(-8 - 16)$$

$$(A \times B) = -6i - 36j - 24k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$i \begin{vmatrix} -36 & -24 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} -6 & -24 \\ 1 & -3 \end{vmatrix} + k \begin{vmatrix} -6 & -36 \\ 1 & 4 \end{vmatrix}$$

$$i(108 + 96) - j(18 + 24) + k(-24 + 36)$$

$$\underline{\underline{(A \times B) \times C = 204i - 42j + 12k}}$$