NAME: AKOMOLEDE MATTHEW GBENGA
DEPARTMENT: PETROLEUM ENGINEERING
MATRIC NO: 19/ENG07/005
COURSE CODE: MAT 104.

1. $\left[\frac{4x^2 - \sin x}{x^3}\right] x \to 0$

$$\lim_{x\to 0}\left[\frac{4x^2 - \sin x}{x^3}\right]$$

By direct sub, we have $\frac{0}{0}$ = Undefined using L'Hopital's Rule, we have

$$\lim_{x\to 0}\left\{\frac{4x^2 - \sin x}{x^3}\right\} = \lim_{x\to 0}\left\{\frac{8x - \cos x}{3x^2}\right\}$$

$$= \lim_{x\to 0}\left\{\frac{8 + \sin x}{6x}\right\} = \lim_{x\to 0}\left\{\frac{\cos x}{6}\right\}$$

$$= \frac{\cos(0)}{6} = \frac{1}{6}$$

2, $y = \frac{7x^2 \cos 8x}{e^{3x}}$

$y = \frac{uv}{w}$

$\frac{dy}{dx} = y\left[\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}\right]$

$u = 7x^2$, $v = \cos 8x$, $w = e^{3x}$

$\frac{du}{dx} = 14x$, $\frac{dv}{dx} = -8\sin 8x$, $\frac{dw}{dx} = 3e^{3x}$

$\frac{dy}{dx} = \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{1}{7x^2}\cdot 14x + \frac{1}{\cos 8x}\cdot -8\sin 8x - \frac{1}{e^{3x}}\cdot 3e^{3x}\right]$

$= \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{2}{x} + \left(-\frac{8\sin 8x}{\cos 8x}\right) - \frac{3e^{3x}}{e^{3x}}\right]$

$= \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x - 3\right]$

③ $y = \cos(5x^2+6x)$

$\frac{dy}{dx} = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{du}{dx} = 10x+6$

$\frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = -\sin u \cdot 10x+6$

$= ~~-\sin u(10x+6)~~$

$= ~~-\sin(5x^2+6x)~~$

$= -(10x+6)\sin u$

$= -(10x+6)\sin(5x^2+6x)$

4, (a) $\int \frac{dx}{(4x+1)}$

(b) $\frac{dx}{(x^2+49)}$

(c) $(e^{6x}+9x^3-\sin 7x+\cos 8x)\,dx$

(d) $x\sqrt{(9+x^2)}\,dx$

Soln

(b) $\int \frac{dx}{x^2+49} = \int \frac{dx}{x^2+7^2}$

$x = 7\tan\theta$

$\frac{dx}{d\theta} = 7\sec^2\theta$

$dx = 7\sec^2\theta\, d\theta$

$x^2+7^2 = 7^2\tan^2\theta + 7^2$

$= 7^2(\tan^2\theta+1)$

$\tan^2\theta + 1 = \sec^2\theta$

$x^2+7^2 = 7^2\sec^2\theta$

$\int \frac{7\sec^2\theta\, d\theta}{7^2\sec^2\theta}$

$\frac{1}{7}\int d\theta$

$\frac{1}{7}\theta + c$

$x = 7\tan\theta$

$\frac{x}{7} = \tan\theta$

$\tan^{-1}\left(\frac{x}{11}\right) = \theta$

$\frac{1}{7}\tan^{-1}\left(\frac{x}{7}\right) + C$

(c) $(e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$

$= \left[\frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C\right]$

$= \frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$

(a) $\int \frac{3dx}{(4x+1)}$

Soln

$u = 4x+1$, then $\int \frac{3dx}{u}$

$\frac{du}{dx} = 4$

$dx = \frac{du}{4}$

$\int \frac{3dx}{(4x+1)} = \int \frac{3du}{4u}$

$= \frac{3}{4}\int \frac{du}{u}$

$= \frac{3}{4}\int u + C$

Scanned with CamScanner

$$= \frac{3}{4}\left[\frac{u^{1+1}}{1+1}\right] + c$$

$$= \frac{3}{4}\left[\frac{u^{2}}{2}\right] + c$$

$$= \frac{3}{4}\left[\frac{(4x+1)}{2}\right] + c$$

$$\int \frac{3\,dx}{(4x+1)} = \frac{3(4x+1)}{8} + c$$

(D) $\cancel{x\sqrt{(3^2+x^2)}\,dx}$ $\quad x\sqrt{(3+x)^2}\,dx$

$\cancel{x\sqrt{(3^2+x^2)}\,dx}$

~~let~~ $\cancel{u = 3^2+x^2}$ $\qquad x(3+x)\,dx$ $\qquad u = 3+x$

$\frac{du}{dx} = 2x$ $\qquad u = 3+x$ $\qquad u - 3 = x$

$\cancel{d[illegible]}$ $\qquad \frac{du}{dx} = 1$ $\qquad x = u-3$

$dx = du$

$$\int x(3+x)\,dx = \int (u-3)u\,du$$

$$= \int (u^2 - 3u)\,du$$

$$= \frac{u^{2+1}}{2+1} - \frac{3u^{1+1}}{1+1} + c$$

Scanned with CamScanner

$$= \frac{u^3}{3} - \frac{3u^2}{2} + C$$

$$= \frac{(3+x)^3}{3} - \frac{3(3+x)^2}{2} + C$$