

$$11) \int x^{1/2} \ln x$$

$$\int x^{1/2} \ln x$$

$$u = x^{1/2} \quad du = \frac{1}{2} x^{-1/2}$$

$$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$$

$$\int v \frac{du}{dx} + \int u \frac{dv}{dx}$$

$$\ln x \int \frac{dx}{2x^{3/2}} + x^{1/2} \int \frac{d \ln x}{dx}$$

$$\ln x \left[\frac{2x^{-3/2}}{-3} \right] + x^{1/2} \left[\frac{1}{2x} \right] + C$$

$$\frac{2 \ln x \cdot (-2) x^{-3/2}}{3} + \frac{x^{1/2}}{2x} + C$$

$$\frac{2 x^{3/2} \ln x}{3} + \frac{\sqrt{x}}{2x} + C$$

$$(2) \int 2 \cos 6t \cos t \, dt = 12 \int \cos 6t \cos t \, dt$$

$$A = 6t \quad B = t$$

$$\begin{aligned} \cos A \cos B &= \frac{1}{2} [\cos(A+B) + \cos(A-B)] \\ &= \frac{1}{2} [\cos(6+t) + \cos(6-t)] \\ &= \frac{1}{2} [\cos 7t + \cos 5t] \\ &= \frac{1}{2} [\cos 7t + \cos 5t] \end{aligned}$$

$$\begin{aligned} \int 2 \cos 6t \cos t \, dt &= \frac{1}{2} \int (2 \cos 7t + \cos 5t) \, dt \\ &= \frac{1}{2} \left[\frac{\sin 7t}{7} - \frac{\sin 5t}{5} \right] \end{aligned}$$

$$= \frac{\sin 7t}{7} - \frac{\sin 5t}{5} + C$$

$$3) \int \sin^3 x \cos^5 x \, dx$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$$

$$\int \cos^5 x \sin^3 x \, dx$$

$$\int u^4 \sin^2 x \frac{-du}{\sin x}$$

$$= \int u^4 \sin^2 x \, -du$$

$$= \int u^4 \sin^2 x \cdot du$$

Recall that

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$= \int \sin^2 x u^4 \, du$$

$$= \int (1 - \cos^2 x) u^4 \, du$$

but $u = \cos x$

$$= \int (1 - u^2) u^4 \, du$$

$$= \int (u^4 - u^6) \, du$$

$$= \left[\frac{u^{4+1}}{4+1} - \frac{u^{6+1}}{6+1} \right] + c$$

$$= \left[\frac{u^5}{5} - \frac{u^7}{7} \right] + c$$

$$= \frac{(\cos x)^5}{5} - \frac{(\cos x)^7}{7} + c$$

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