

- ① A particle moves along a curve $x = 8t^3$, $y = 4t^3 - 7t$ and $z = t + 3$ where t is time. Find its
i) velocity ii) acceleration.

$$r = xi + yj + zk.$$

$$r = 8t^3i + (4t^3 - 7t)j + (t + 3)k.$$

$$\text{velocity vector } (v) = \frac{dr}{dt}.$$

$$v = 24t^2i + (12t^2 - 7)j + k.$$

$$\text{acceleration vector} = \frac{dv}{dt}$$

$$a = 48ti + (24t)j$$

$$|a| = \sqrt{48^2 + 24^2} \quad t = 1$$

$$|a| = 48(1) + 24(1)$$

$$|a| = 48i + 24j$$

$$|a| = \sqrt{48^2 + 24^2}$$

$$|a| = 53.66 \text{ms}^{-2}.$$

$$v = 24i + 5j + k$$

$$|v| = \sqrt{24^2 + 5^2 + 1^2}$$

$$= 24.51 \text{ms}^{-1}$$

- ② Unit tangent vector.

$$r = xi + yj + zk.$$

$$r = 3ti + t^3j + t^2k$$

$$\frac{dr}{dt} = 3i + 3t^2j + 2tk \quad \text{at } t = 1$$

$$\left| \frac{dr}{dt} \right| = \sqrt{3^2 + 3^2 + 2^2} \quad \frac{\vec{r}}{\|\vec{r}\|} = \frac{(3, 3, 2)}{\sqrt{22}}$$

$$= \sqrt{22}.$$

$$= \frac{3}{\sqrt{22}}, \frac{3}{\sqrt{22}}, \frac{2}{\sqrt{22}} \quad \text{at } t = 1$$