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COMPUTER ENGINEERING (19/ENG02/054)
MAT104 ASSIGNMENT
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Integrate the following with respect to their variables
1) $x^{1/2}\ln x$
2) $2\cos 6t \cos t$
3) $\sin^3 x \cos^4 x$

ANSWER

1) $dy/dx = x^{1/2}\ln x$

$\int dy = \int x^{1/2}\ln x \, dx$

Integrate by parts $\int fg' = fg - \int f'g$

where $g' = x^{1/2}$ $\therefore$ $g = \frac{2x^{3/2}}{3}$

$f = \ln(x)$ $\therefore$ $f' = 1/x$

from the formula, $\int x^{1/2}\ln x = \frac{2x^{3/2}\ln x}{3} - \int \frac{2x^{1/2}}{3} dx$

Solving for $\int \frac{2x^{1/2}}{3} dx$

we have $\Rightarrow$ $\frac{2}{3}\int x^{1/2} dx$

$= \frac{2}{3}\left[\frac{2x^{3/2}}{3}\right]$

$\int \frac{2x^{1/2}}{3} dx = \frac{4x^{3/2}}{9}$

Put the Integral into the formula

$\int x^{1/2}\ln x = \frac{2x^{3/2}\ln x}{3} - \frac{4x^{3/2}}{9}$

$\int x^{1/2}\ln x = \frac{6x^{3/2}\ln x - 4x^{3/2}}{9}$

$\int x^{1/2}\ln x = \frac{2x^{3/2}(3\ln(x) - 2)}{9} + C$

(2) $2\cos 6(t)\cos(t)$

$$dy/dt = 2\cos 6(t)\cos(t)$$

$$\int dy = \int 2\cos t\cos 6t \, dt \quad \text{(Rearranged)}$$

Apply linearity: $\int dy = 2\int \cos t\cos 6t \, dt$

Apply product-to-sum formula:

$$\cos(x)\cos(y) = \tfrac{1}{2}\left(\cos(y+x) + \cos(y-x)\right)$$

$$\therefore \cos t\cos 6t = \tfrac{1}{2}\left(\cos(6t+t) + \cos(6t-t)\right)$$

$$\cos t\cos 6t = \tfrac{1}{2}\left(\cos 7t + \cos 5t\right)$$

$$\therefore \int dy = 2\int \frac{\cos 7t + \cos 5t}{2} \, dt$$

Apply linearity again: $\int dy = \frac{\cancel{2}}{\cancel{2}}\int \cos 7t + \cos 5t \, dt$

$$\int dy = \left(\int \cos 7t + \int \cos 5t\right) dt$$

Integrate $\cos 7t$

$\Rightarrow$ let $u = 7t$; $du/dt = 7$

$dt = du/7$

$\therefore \int \cos 7t \, dt$

$\int \cos u \, du/7$

$1/7 \int \cos u \, du$

$1/7 \sin u$

Integrate $\cos 5t$

$\Rightarrow$ let $u = 5t$; $du/dt = 5$ $\therefore$ $dt = du/5$

$\int \cos u \, dt$

$\int \cos \, du/5$

$1/5 \int \cos u \, du$

$1/5 \sin u$

$$\cancel{\int dy} \; y = \left[\frac{\sin u}{7} + \frac{\sin u}{5}\right] + C$$

Recall that $u = 7t$ and $5t$

$$\therefore y = \frac{\sin 7t}{7} + \frac{\sin 5t}{5} + C$$

3) $\frac{dy}{dx} = \sin^3 x \cos^4 x$

$\int dy = \int \sin^3 x \cos^4 x \, dx$

but $\sin^2 x = 1 - \cos^2 x$

Rearrange $\Rightarrow$ $\int dy = \int \cos^4 x \sin^3 x \, dx$

$\int dy = \int \cos^4 x (1 - \cos^2 x) \sin x \, dx$

let $u = \cos x$

$\therefore du/dx = -\sin x$; $dx = \frac{du}{-\sin x}$

$\int dy = \int u^4 (1 - u^2) \sin x \, dx$

$\int dy = \int -u^4 (u^2 - 1) \sin x \, dx$

$\int dy = -\int u^4 (u^2 - 1) \sin x \, dx$

but $dx = -\frac{du}{\sin x}$

$\therefore \int dy = \cancel{+}\int u^4 (u^2 - 1) \cancel{\sin x} \cdot \frac{du}{\cancel{+}\cancel{\sin x}}$

$\int dy = \int u^4 (u^2 - 1) \, du$

$\int dy = \int (u^6 - u^4) \, du$

$\int dy = \left(\int u^6 - \int u^4\right) du$

$y = \left[\frac{u^{6+1}}{(6+1)} - \frac{u^{4+1}}{(4+1)}\right] + c$

$y = \frac{u^7}{7} - \frac{u^5}{5} + c$

Recall that $u = \cos x$

$\therefore$ $y = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + c$