

NAME: ISAAC ENE GRACE
DEPARTMENT: BIOMEDICAL ENGINEERING
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MAT 102 ASSIGNMENT

1. A particle moves along a curve $x = 8t^3$, $y = 4t^3 - 7t$ and $z = t + 3$, where t is time. Find it's:

(i) velocity.

(ii) acceleration.

Solution

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = (8t^3)\hat{i} + (4t^3 - 7t)\hat{j} + (t + 3)\hat{k}$$

$$\begin{aligned} \text{(i) velocity, } \vec{v} &= \frac{d\vec{r}}{dt} = \frac{d}{dt} [(8t^3)\hat{i} + (4t^3 - 7t)\hat{j} + (t + 3)\hat{k}] \\ &= (24t^2)\hat{i} + (12t^2 - 7)\hat{j} + \hat{k} \end{aligned}$$

$$\begin{aligned} \text{(ii) acceleration, } \vec{a} &= \frac{d\vec{v}}{dt} = \frac{d}{dt} [(24t^2)\hat{i} + (12t^2 - 7)\hat{j} + \hat{k}] \\ &= (48t)\hat{i} + (24t)\hat{j} + 0 \\ &= (48t)\hat{i} + (24t)\hat{j} \end{aligned}$$

2. Find the unit tangent vector to the space curve $x = 3t$, $y = t^3$ and $z = t^2$ at $t = 1$

Solution

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{r} = (3t)\hat{i} + (t^3)\hat{j} + (t^2)\hat{k} \text{ at the point } t = 1$$

Tangent vector,

$$\vec{r}'(t) = (3\hat{i}) + (3t^2)\hat{j} + (2t)\hat{k}$$

$$\vec{r}'(1) = 3\hat{i} + 3(1)^2\hat{j} + 2(1)\hat{k}$$

$$\vec{r}'(1) = 3\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{r}'(1) = (3, 3, 2)$$

$$\text{Unit tangent vector: } \frac{\vec{v}}{|\vec{v}|} = \frac{(3, 3, 2)}{\sqrt{3^2 + 3^2 + 2^2}} = \frac{(3, 3, 2)}{\sqrt{22}}$$

$$\left(\frac{3}{\sqrt{22}}, \frac{3}{\sqrt{22}}, \frac{2}{\sqrt{22}} \right)$$

$$\left(\frac{3\sqrt{22}}{22}, \frac{3\sqrt{22}}{22}, \frac{2\sqrt{22}}{22} \right)$$