

NAME: ISAAC ENG GRACE
DEPARTMENT: BIOMEDICAL ENGINEERING
MATRIC NUMBER: 19108081004

MAT 104 ASSIGNMENT

Integrate the following:
1. $x^{1/2} \ln x$

Solution

$$\int x^{1/2} \ln x \, dx = \int u \, dv = uv - \int v \, du$$

$$u = \ln(x), \, du = \frac{1}{x} \, dx$$

$$dv = x^{1/2}, \, v = \int x^{1/2} \, dx = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2}$$

Recall:

$$\int u \, dv = \ln(x) \cdot \frac{2}{3} x^{3/2} - \int \frac{2}{3} x^{3/2} \cdot \frac{1}{x} \, dx = \frac{2}{3} x^{3/2} \ln(x) - \frac{2}{3} x^{1/2+1}$$

$$= \frac{2}{3} x^{3/2} \ln(x) - \frac{2}{3} \cdot \frac{x^{3/2}}{3/2} = \frac{2}{3} x^{3/2} \ln(x) - \frac{4}{9} x^{3/2}$$

$$= \frac{2}{3} \sqrt{x^3} \cdot (\ln(x) - 2/3) + c$$

2. $2 \cos 6t \cos t \, dt$

Solution.

$$\int 2 \cos(6t) \cos(t) \, dt = 2 \int \cos(6t) \cos(t) \, dt$$

Recall:

$$\cos A \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$$

$$= 2 \int \frac{1}{2} [\cos(6-1)t + \cos(6+1)t] \, dt$$

$$= 2 \times \frac{1}{2} \int [\cos(5t) + \cos(7t)] \, dt$$

$$= \int \cos(5t) + \cos(7t) \, dt$$

$$= \int \cos(5t) \, dt + \int \cos(7t) \, dt$$

$$= \frac{\sin(5t)}{5} + \frac{\sin(7t)}{7} + c$$

$$3. \sin^3 x \cos^4 x$$

Solution

$$\int \sin^3(x) \cos^4(x) dx = \int \sin(x) \cdot \sin^2(x) \cdot \cos^4(x) dx \\ = \int \sin(x) [1 - \cos^2(x)] \cdot \cos^4(x) dx$$

$$\text{Let } u = \cos(x)$$

$$\frac{du}{dx} = -\sin(x) \Rightarrow -du = \sin(x) dx$$

$$= \int \sin(x) [1 - u^2] u^4 dx = \int (1 - u^2) u^4 \sin(x) dx$$

$$= \int (1 - u^2) u^4 \cdot -du$$

$$= \int (u^6 - u^4) du = \frac{u^7}{7} - \frac{u^5}{5} + C$$

$$= \frac{\cos^7(x)}{7} - \frac{\cos^5(x)}{5} + C$$