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1. If $\vec{A} = 3i + 7j - 2k$, $\vec{B} = i + 3j + 7k$, $\vec{C} = 9i - 4j + 6k$, find the angle between (i) $\vec{A}$ and $\vec{C}$ (ii) $\vec{B}$ and $\vec{C}$ (iii) the unit vector in the direction of $(\vec{A} + \vec{B} + \vec{C})$.

Solution

i. $\vec{A} \cdot \vec{C} = |\vec{A}||\vec{C}| \cos\theta$

$$\cos\theta = \frac{\vec{A} \cdot \vec{C}}{|\vec{A}| \cdot |\vec{C}|} \quad ; \quad \theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{C}}{|\vec{A}| \cdot |\vec{C}|}\right)$$

$$\vec{A} \cdot \vec{C} = (3i + 7j - 2k) \cdot (9i - 4j + 6k)$$
$$= 27 - 28 - 12$$
$$\vec{A} \cdot \vec{C} = -13$$

$$|\vec{A}| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{62}$$
$$|\vec{C}| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\therefore \theta = \cos^{-1}\left(\frac{-13}{\sqrt{62} \cdot \sqrt{133}}\right) = 98.2^0$$

ii. $\vec{B} \cdot \vec{C} = (i + 3j + 7k) \cdot (9i - 4j + 6k)$

$$\vec{B} \cdot \vec{C} = 9 - 12 + 42 = 39$$
$$|\vec{B}| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$
$$|\vec{C}| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$\therefore \theta = \cos^{-1}\left(\frac{\vec{B} \cdot \vec{C}}{|\vec{B}| \cdot |\vec{C}|}\right) = \cos^{-1}\left(\frac{39}{\sqrt{59} \times \sqrt{133}}\right)$$

$$\theta = 63.9^0$$

4. $\vec{A} + \vec{B} + \vec{C} = (5i + 7j - 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$

$\vec{A} + \vec{B} + \vec{C} = 13i + 6j + 11k$

$|(\vec{A} + \vec{B} + \vec{C})| = \sqrt{13^2 + 6^2 + 11^2} = \sqrt{326} = 18.1$

$\hat{e}_{(\vec{A}+\vec{B}+\vec{C})} = \dfrac{(\vec{A} + \vec{B} + \vec{C})}{|(\vec{A} + \vec{B} + \vec{C})|}$

$\hat{e}_{(\vec{A}+\vec{B}+\vec{C})} = \dfrac{13i + 6j + 11k}{18.1}$

2. A particle moves along a curve, $x = 8t^2$, $y = t^2 - 4t$, $z = t+1$ where t is time. Find the modulus of acceleration at $t = 1$

Solution

$r = xi + yj + zk$

$r = (8t^2)i + (t^2 - 4t)j + (t+1)k$

$V = \dfrac{dr}{dt} = (16t)i + (2t - 4)j + k$

$A = \dfrac{dv}{dt} = \dfrac{d^2r}{dt^2} = 16i + 2j + 0k$

$|A|_{t=1} = \sqrt{16^2 + 2^2 + 0^2}$

$= \sqrt{256 + 4} = \sqrt{260}$

$|A| = 16.12\ m/s^2$

3. If $\vec{A} = 4i + 2j - 4k$, $\vec{B} = 8i - 2j + k$, $\vec{C} = i + 4j - 3k$, find the vector triple product $(A \times B) \times C$

Solution

$$A \times B = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - j\begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k\begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$= i(2-8) - j(4+32) + k(-8-16)$$

$$A \times B = -6i - 36j - 24k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= i\begin{vmatrix} -36 & -24 \\ 4 & -3 \end{vmatrix} - j\begin{vmatrix} -6 & -24 \\ 1 & -3 \end{vmatrix} + k\begin{vmatrix} -6 & -36 \\ 1 & 4 \end{vmatrix}$$

$$= i(108+96) - j(18+24) + k(-24+36)$$

$$(A \times B) \times C = 204i - 42j + 12k$$