1, $x^{1/2}\ln x$

2, $2\cos 6t \cos t$

3, $\sin^3 x \cos^4 x$

Solution

1, $x^{1/2}\ln x$

$\int x^{1/2}\ln x$

$u = x^{1/2} \quad dv = \ln x$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\int v\frac{du}{dx} + \int u\frac{dv}{dx}$

$\ln x\int \frac{dx^{1/2}}{dx} + x^{1/2}\int\frac{d\ln x}{dx}$

$\ln x\left[\frac{x^{3/2}}{3/2}\right] + x^{1/2}\left[\int\frac{1}{x}\right] + c$

$\frac{2\ln x}{3}\cdot x^{3/2} + \frac{x^{1/2}}{x} + c$

$\frac{2x^{3/2}\ln x}{3} + \frac{\sqrt{x}}{x} + c$

2) $\int 2\int \cos 6t \cos t\, dt = 12\int \cos 6t \cos t\, dt$

let $A = 6t \quad B = 1t$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= \frac{2}{2}[\cos(6+1) + \cos(6-1)]$

$= \frac{2}{2}[\cos 7t + \cos 5t]$

~~$= \frac{1}{2}[$~~

$\int 2\cos 6t \cos t\, dt = \frac{1}{2}\int(2\cos 7t + \cos 5t)$

$= \frac{1}{2}\left[\frac{2\sin 7t}{7} - \frac{\sin 5t}{5}\right]$

$= \frac{\sin 7t}{7} - \frac{\sin 5t}{5} + c$

3) $\int \sin^3 x \cos^4 x \, dx$

$U = \cos x$

$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$

$\int \cos^4 x \sin^3 x \, dx$

$\int U^4 \cancel{\sin^3 x} \; \frac{-du}{\sin x}$

$= \int U^4 \sin^2 x \; -du$

$- \int U^4 \sin^2 x \cdot du$

Recall that

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$- \int \sin^2 x \, U^4 \cdot du$

$- \int (1 - \cos^2 x) U^4 \, du$

but $U = \cos x$

$- \int (1 - U^2) U^4 \, du$

$- \int (U^4 - U^6) \, du$

$- \left[\frac{U^{4+1}}{4+1} - \frac{U^{6+1}}{6+1} \right] + c$

$- \left[\frac{U^5}{5} - \frac{U^7}{7} \right] + c$

$\frac{(\cos x)^7}{7} - \frac{(\cos x)^5}{5} + c$

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