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19/ENG05/011 100 lvl
Mat 102

1) $x = 7t^2$, $y = 6t^2 - 4t$, $z = t - 5$

$V_x = \frac{dx}{dt} = 14t$

$V_y = \frac{dy}{dt} = 12t - 4$

$V_z = 1 - 0 = 1$

$\therefore$ Velocity $= V_x + V_y + V_z$
$= 14t + 12t - 4 + 1$
$= 26t - 3$

2) ~~A = i + 2j~~ $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$.

$$(B \times C) = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$(B \times C) = |(-3 \times -3) - (4 \times 1)|i - |(-3 \times 2) - (0 \times 1)|j + |(4 \times 2) - (-3 \times 0)|k$$
$$= |9 - 4|i - |-6 - 0|j + |8 + 0|k$$
$$= 5i + 6j + 8k.$$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$A \times (B \times C) = |(8 \times 2) - (-4 \times 6)|i - |(8 \times 1) - (-4 \times 5)|j + |(6 \times 1) - (5 \times 2)|$$
$$= |16 + 24|i - |8 + 20|j + |6 - 10|k$$
$$= 40i - 28j - 4k$$

3 $R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$

$\int R = (\int 4 \times \int \sin 3t) i + (\int 4 \times \int e^{3t}) j + (\int 7t^3) k$

$\int R = (4t \times -\cos 3t) i + \left(4t \times \frac{1}{3} e^{3t}\right) j + \left(\frac{7t^{3+1}}{3+1}\right)$

$\therefore \int R = (-4t\cos 3t + c) i + \left(\frac{4te^{3t}}{3} + c\right) j + \left(\frac{7t^4}{4} + c\right) k$

4) $A = 7i + 2j - k, \quad B = 2i + j + 4k \quad , C = i + j + k$

$(A + C) \cdot (B - A) = ?$

$A + C = (7i + i) + (2j + j) + (k - k)$

$= 8i + 3j$

$B - A = (2i - 7i) + (j - 2j) + (4k - (-k))$

$= -5i - j + 5k$

$\therefore (A + c) \cdot (B - A) = (8 \times -5) i + (3 \times -1) j + (0 \times 5) k$

$= -40i - 3j$

5) $x = t,\ y = t^2,\ z = t^3 \quad t = 1$

let $r = (t, t^2, t^3)$

$r' = \frac{dr}{dt} = (1, 2t, 3t^2)$

$|r'| = \sqrt{(1)^2 + (2t)^2 + (3t^2)^2} = \sqrt{1 + 4t^2 + 9t^4}$

$\hat{T} = \frac{r'}{|r'|} = \frac{1}{\sqrt{1 + 4t^2 + 9t^4}} (1, 2t, 3t^2)$

from $t = 1$

$\hat{T} = \frac{1}{\sqrt{1 + 4(1)^2 + 9(1)^4}}$ ~~$(1, 2t, 3t^2)$~~ $(1, 2(1), 3(1)^2)$

$= \frac{1}{\sqrt{1+4+9}} (1, 2, 3) = \frac{1}{\sqrt{14}} (1, 2, 3) = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right) = \left(\frac{1\sqrt{14}}{14}, \frac{\sqrt{14}}{7}, \frac{3\sqrt{14}}{14}\right)$