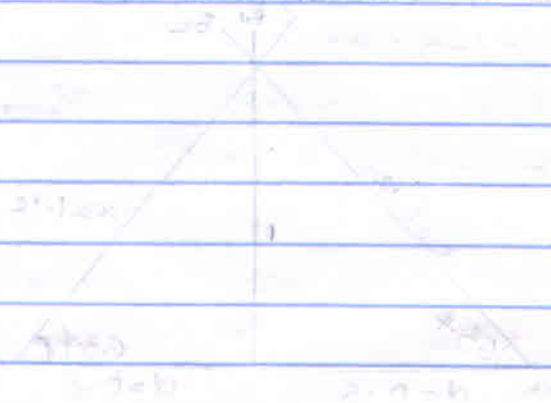


1. Charging by Induction:

Electric charges can be obtained on an object without touching it, by a process called electrostatic induction.

Consider a positively charged rubber rod brought near a neutral (uncharged) conducting sphere that is insulated so that there is no conducting path to ground as shown below. The repulsive force between the protons in the rod and those in the sphere causes a redistribution of charges on the sphere so that some protons move to the side of the sphere farthest away from the rod. The region of the sphere nearest to positively charged rod has an excess of negatively charged because of the migration of protons away from this location. If a grounded conducting wire is then connected to the sphere, as in some of the protons leave the sphere and travel to the earth. If the wire to ground is then removed, the conducting sphere is left with an excess of induced negative charge.

Finally, when the rubber rod is removed from the vicinity of the sphere, the induced negative charge remains on the ungrounded sphere and becomes uniformly distributed over the surface of the sphere.



16) $k = 9 \times 10^9$

$q_1 + q_2 = 5 \times 10^{-5} \text{ C}$

$F = 1 \text{ N}$

$d = 2 \text{ m}$

Calculate the charge on each sphere

$k = 9 \times 10^9$

$F = \frac{k q_1 q_2}{r^2}$

$1 = \frac{9 \times 10^9 \times (q_1 q_2 \cdot 5 \times 10^{-5})}{d^2}$

$4 = 9 \times 10^9 \times 5 \times 10^{-5} q_1 + 9 \times 10^9 q_2$

$4 = 4.5 \times 10^5 q_2 + 9 \times 10^9 q_2$

It is a quadratic equation.

$9 \times 10^9 q_2^2 - 4.5 \times 10^5 q_2 + 4 = 0$

$q_1 = 0.000011 \text{ C}$

$q_2 = 0.000038 \text{ C}$

$\approx q_1 = 1.1 \times 10^{-5} \text{ C}$

$q_2 = 3.8 \times 10^{-5} \text{ C}$

1e) $Q_1 = Q_2 = 8 \mu\text{C}$

$d = 0.5 \text{ m}$

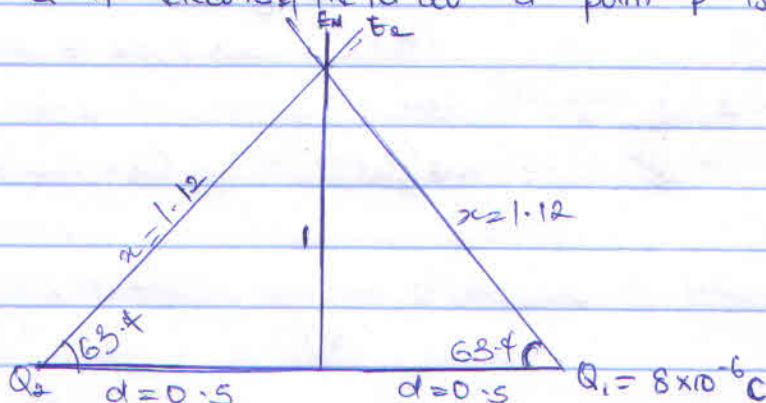
determine Q if electric field at a point p is zero

$\tan \theta = \frac{\text{opp}}{\text{adj}}$

$\tan \theta = \frac{1}{0.5} = 2$

$\theta = \tan^{-1} 2$

$\theta = 63.4^\circ$



$x^2 = 1^2 + 0.5^2$

$\sqrt{x} = \sqrt{1.25}$

$x = \sqrt{1.25}$

$x = 1.12$

$$E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 5739.795918$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(1.12)^2} = 5739.795918$$

$$E_q = \frac{kq}{r^2} = \frac{9 \times 10^9 \times q}{1} = 9 \times 10^9 q$$

| Vector | angle | x-comp | y-comp |
|-------------------------|--------------|--|--|
| $E_1 = 5739.795918$ | 63.4° | $E_1 \times \cos \theta$ -2570.045785 | 5132.262839 |
| $E_2 = 5739.795918$ | 63.4° | 2570.045785 | 5132.262839 |
| $E_q = 9 \times 10^9 q$ | 90 | $E_q \cos \theta = 0$ $E_x = 0$ | $9 \times 10^9 q$ $E_y = 10264.52568$ |

$$\text{magnitude} = \sqrt{E_x^2 + E_y^2}$$

$$E_q = \sqrt{(0)^2 + (10264.52568)^2}$$

$$\text{Since } E = 0$$

$$0 = 9 \times 10^9 q + 10264.52568$$

making q subject of formula

$$q = \frac{-10264.52568}{9 \times 10^9}$$

$$q = 1.140502855 \times 10^{-6}$$

$$q = 1.141 \mu\text{C}$$

3a Volume charge density: $\rho = \frac{dQ}{dV} \rightarrow dQ = \rho dV$

Surface charge density: $\sigma = \frac{dQ}{dA} \rightarrow dQ = \sigma dA$

Linear charge density: $\lambda = \frac{dQ}{dL} \rightarrow dQ = \lambda dL$

b) ELECTRIC POTENTIAL DIFFERENCE

The electric potential difference between two points in an electric field can be defined as the work done per unit charge against electrical forces when a charge is transported from one point to the other. It is measured in Volt (V) or Joules per Coulomb (J/C). Electric potential difference is a scalar quantity.

Consider the diagram above, Suppose a test charge q_0 is moved from point A to point B along an arbitrary path inside an electric field E . The electric field E exerts a force $F = q_0 E$ on the charge as shown above. To move the test charge from A to B at constant velocity, an external force of $F = -q_0 E$ must act on the charge. Therefore, the elemental work done dW is given as

$$dW = F \cdot dl \quad (1)$$

But

$$F = -q_0 E \quad (2)$$

Substituting eqn (2) in (1) gives:

$$dW = -q_0 E dl \quad (3)$$

Then total work done in moving the test charge from A to B is:

$$W(A \rightarrow B)_{q_0} = -q_0 \int_A^B E dl \quad (4)$$

from the definition of electric potential difference, it follows that

$$V_B - V_A = \frac{W(A \rightarrow B)_{q_0}}{q_0} \quad (5)$$

Putting eqn (4) in (5) yields

$$V_B - V_A = - \int_A^B E dl \quad (6)$$

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19/ENG05/011

PHY 102 Assignment Continuation:

4a Magnetic flux is defined as the strength of the magnetic field which can be represented by line of forces. It is represented by the symbol Φ mathematically given by $\Phi = B \cdot dA$.

4b $\omega = v/r = qB/m$

$m = 9.11 \times 10^{-31} \text{ kg}$

$r = 1.4 \times 10^{-9} \text{ m}$

$B = 3.5 \times 10^{-1} \text{ tesla}$

$\omega = qB/m_e \cdot m_e = 1.6 \times 10^{-19} \text{ C}$

where $m_e = \text{mass of electron} = 1.6 \times 10^{-19} \text{ kg}$

$$\omega = \frac{(1.6 \times 10^{-19}) \times (3.5 \times 10^{-1})}{(9.11 \times 10^{-31})} \text{ T}^{-1} \times \text{C} = 6.15 \times 10^{10}$$

$\omega = 6.15 \times 10^{10} \text{ rad/s}$

4c (i) mass of the electron = $9.11 \times 10^{-31} \text{ kg}$

(ii) A radius of $1.4 \times 10^{-9} \text{ m}$.

(iii) magnetic field of $3.5 \times 10^{-1} \text{ weber / meter square}$.

To find Cyclotron frequency which is the same as angular speed.

Recall that angular speed is given as $\omega = \frac{v}{r} = \frac{qB}{m}$

Substituting we have $\omega = \frac{v}{r} = \frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}}$

$$\frac{qB}{m} = \frac{1.6 \times 10^{-19} \times 3.5 \times 10^{-1}}{9.11 \times 10^{-31}} = 6.15 \times 10^{10} \text{ rad/s}$$

So since cyclotron frequency is equal to angular speed the cyclotron frequency is equal to $6.15 \times 10^{10} \text{ rad/s}$, having a unit as $\frac{1}{T}$ which is equal to the unit of frequency dimensionally.

5a) Biot-Savart law states that the magnetic field is directly proportional to the product permeability of free space (μ_0), the current (I), the change in length, the radius and inversely proportional to square of radius (r^2). It can be represented mathematically by

$$dB = \frac{\mu_0 I dl \times r}{4\pi r^2}$$

where μ_0 is a constant called permeability of free space:

$$\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$$

The unit of B is weber / metre square.

5b) Magnetic field of a straight current carrying conductor

Applying the Biot-Savart law, we find magnitude of the field dB

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin \phi}{r^2}$$

$$\sin(\pi - \phi) = \sin \theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{r^2}$$

from diagram, $r^2 = x^2 + y^2$ (Pythagoras theorem)

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{dl \sin(\pi - \phi)}{x^2 + y^2} \quad \dots \textcircled{1}$$

$$\text{But } \sin(\pi - \phi) = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{(x^2 + y^2)^{1/2}} \quad \dots \textcircled{2}$$

Substituting eqn ② into ① we have.

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)(x^2 + y^2)^{1/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a dl \frac{x}{(x^2 + y^2)^{3/2}}$$

Recall $dl = dy$

$$B = \frac{\mu_0 I}{4\pi} \int_{-a}^a \frac{x}{(x^2 + y^2)^{3/2}} dy$$

$$B = \frac{\mu_0 I x}{4\pi} \int_{-a}^a \frac{1}{(x^2 + y^2)^{3/2}} dy \quad \dots \textcircled{3}$$

Using Special Integrals:

$$\int \frac{dy}{(x^2 + y^2)^{3/2}} = \frac{y}{x^2 (x^2 + y^2)^{1/2}}$$

Equation (3) therefore becomes

$$B = \frac{\mu_0 I x}{4\pi} \left[\frac{2y}{x^2(x^2+y^2)^{3/2}} \right]_0^a$$

$$B = \frac{\mu_0 I x}{4\pi} \left(\frac{2a}{x^2(x^2+a^2)^{3/2}} \right)$$

$$B = \frac{\mu_0 I}{4\pi x} \left(\frac{2a}{(x^2+a^2)^{3/2}} \right)$$

when the length $2a$ of the conductor is very great in comparison to its distance x from the point P , we consider it infinitely long, that is, when a is much larger than x ,

$$(x^2+a^2)^{3/2} \approx a^3, \text{ as } a \rightarrow \infty$$

$$\therefore B = \frac{\mu_0 I}{2\pi x}$$

In a physical situation, we have axial symmetry about the y -axis, thus at all points in a circle of radius r around the conductor the magnitude of B is:

$$B = \frac{\mu_0 I}{2\pi r} \quad (**)$$

Equation (**) defines the magnitude of the magnetic field or flux density B near a long straight current carrying conductor.