

$$1. A = 3i + 7j - 2k, B = i + 8j + 7k, C = 9i - 4j + 6k$$

Soln

i) Angle between A and C

$$A \times C = \begin{vmatrix} i & j & k \\ 3 & 7 & -2 \\ 9 & -4 & 6 \end{vmatrix}$$

$$= i(42 - (-8)) - j(18 - (-18)) + k(12 - 63)$$

$$= i(42 + 8) - j(18 + 18) + k(12 - 63)$$

$$A \times C = 50i - 36j - 51k$$

$$|A \times C| = \sqrt{50^2 + (-36)^2 + (-51)^2}$$

$$= \sqrt{6397}$$

$$= 79.98$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{62}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$$

$$|A| \cdot |C| = \sqrt{62} \times \sqrt{133}$$

$$= 90.81$$

Recall, $|\bar{A} \times \bar{C}| = |A| \cdot |C| \sin \theta$

$$\sin \theta = \frac{|\bar{A} \times \bar{C}|}{|A| \cdot |C|} = \frac{79.98}{90.81}$$

$$\theta = \sin^{-1} \left(\frac{79.98}{90.81} \right)$$

$$\theta = 61.73^\circ$$

\therefore the angle between A and C is 61.73°

$$\text{iii. } (A+B+C)$$

$$= (3i + 7j - 2k) + (i + 8j + 7k) + (9i - 4j + 6k)$$

$$= 13i + 6j + 11k$$

$$\text{Recall, } \hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$|A+B+C| = \sqrt{13^2 + 6^2 + 11^2}$$
$$= \sqrt{326}$$

$$\text{Unit vector of } (A+B+C) = \frac{13i + 6j + 11k}{\sqrt{326}}$$

$$= \frac{13i}{\sqrt{326}} + \frac{6j}{\sqrt{326}} + \frac{11k}{\sqrt{326}}$$

$$2. \quad x = -8t^2, \quad y = t^2 - 4t, \quad z = t + 1$$

$$r = (-8t^2)i + (t^2 - 4t)j + (t + 1)k$$

$$\frac{dr}{dt} = (-16t)i + (2t - 4)j + k$$

$$\frac{d^2r}{dt^2} = -16i + 2j$$

$$\left| \frac{d^2r}{dt^2} \right| = \sqrt{(-16)^2 + (2)^2}$$

$$= \sqrt{260} = 2\sqrt{65} \text{ or } 16.12$$

\therefore The modulus is 16.12

iii. B and C

$$B \times C = \begin{vmatrix} i & j & k \\ 1 & 3 & 7 \\ 9 & -4 & 6 \end{vmatrix}$$

$$= i(18 - (-28)) - j(6 - 63) + k(-4 - 27)$$

$$= i(18 + 28) - j(6 - 63) + k(-4 - 27)$$

$$= 46i + 57j - 31k$$

~~$|\theta| = \sqrt{\quad}$~~

$$|B \times C| = \sqrt{46^2 + 57^2 + (-31)^2} = \sqrt{6326}$$

$$= 79.54$$

$$|B| = \sqrt{1^2 + 3^2 + 9^2} = \sqrt{59} = 7.68$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133} = 11.53$$

$$|B| \cdot |C| = 7.68 \times 11.53 = \sqrt{59} \times \sqrt{133}$$

$$= \cancel{88.58} 88.58$$

Recall, $|B \times C| = \sin \theta |B| |C|$

$$\sin \theta = \frac{|B \times C|}{|B| \cdot |C|} = \frac{79.54}{88.58}$$

$$\sin \theta = \sin^{-1} \left(\frac{79.54}{88.58} \right)$$

$$\theta = 63.89^\circ$$

\therefore Angle between B and C is 63.89°

$$3. A = 4i + 2j - 4k, B = 8i - 2j + k, C = i + 4j - 3k,$$

$$(A \times B) \times C.$$

$$A \times B = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$= i(2-8) - j(4 - (-32)) + k(4 - 16)$$

$$= i(2-8) - j(4 + 32) + k(4 - 16)$$

$$= -6i - 36j - 12k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -12 \\ 1 & 4 & -3 \end{vmatrix}$$

$$= i(108 - (-48)) - j(18 - (-12)) + k(-24 - (-36))$$

$$= i(108 + 48) - j(18 + 12) + k(-24 + 36)$$

$$= 156i - 30j + 12k$$