[redacted] 19|ENG05|048

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MECHATRONICS ENGINEERING

MAT 102 ASSIGNMENT

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(1) $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$,

find the angle between:

(a) A and C; Using their dot product:

$$\bar{A} \cdot \bar{C} = |A||B| \cos\theta$$

$$\therefore \cos\theta = \frac{\bar{A} \cdot \bar{C}}{|A||B|}$$

$$\bar{A} \cdot \bar{C} = (3i + 7j - 2k) \cdot (9i - 4j + 6k)$$

$$= (3 \times 9) + (7 \times -4) + (-2 \times 6)$$

$$= 27 - 28 - 12$$

$$\bar{A} \cdot \bar{C} = -13$$

$$|A| = \sqrt{3^2 + 7^2 + (-2)^2} = \sqrt{9 + 49 + 4} = \sqrt{62}$$

$$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$$

$$\therefore \cos\theta = \frac{-13}{\sqrt{62} \times \sqrt{133}}$$

$$\cos\theta = \frac{-13}{90.8075}$$

$$\cos\theta = -0.1432$$

$$\theta = \cos^{-1}(-0.1432)$$

$$\theta = 98.23^\circ$$

(b) $\bar{B}$ and $\bar{C}$; Using their dot product

$\bar{B} \cdot \bar{C} = |B||C| \cos\theta$

$\therefore \cos\theta = \dfrac{\bar{B} \cdot \bar{C}}{|B||C|}$

$\bar{B} \cdot \bar{C} = (i + 3j + 7k) \cdot (9i - 4j + 6k)$

$\bar{B} \cdot \bar{C} = (1 \times 9) + (3 \times -4) + (7 \times 6)$

$\bar{B} \cdot \bar{C} = 9 - 12 + 42$

$\bar{B} \cdot \bar{C} = 39$

$|B| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{1 + 9 + 49} = \sqrt{59}$

$|C| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{81 + 16 + 36} = \sqrt{133}$

$\therefore \cos\theta = \dfrac{39}{\sqrt{59} \times \sqrt{133}} = \dfrac{39}{88.58}$

$\therefore \cos\theta = \dfrac{39}{88.58} = 0.4403$

$\theta = \cos^{-1}(0.4403)$

$\theta = 63.88°$

(c) Unit vector in the direction of $(A + B + C)$

$A + B + C = (3i + 7j - 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$

$= [3i + i + 9i] + [7j + 3j - 4j] + [-2k + 7k + 6k]$

$A + B + C = 13i + 6j + 11k$

$|A + B + C| = \sqrt{13^2 + 6^2 + 11^2}$

$|A + B + C| = \sqrt{326} = 18.06$

$\therefore \hat{e}_{(A+B+C)} = \dfrac{A + B + C}{|A + B + C|} = \dfrac{13i + 6j + 11k}{18.06}$ or $\dfrac{13i}{18.06} + \dfrac{6j}{18.06} + \dfrac{11k}{18.06}$

(2) $x = 8t^2$, $y = t^2 - 4t$, $z = t + 1$; where t is time. find the modulus of acceleration at $t = 1$

Acceleration $= \frac{dV}{dt} = \frac{d^2r}{dt^2}$

$r = xi + yj + zk$

~~$r = (8t^2)i + (t^2)j$~~.

$r = (8t^2)i + (t^2 - 4t)j + (t+1)K$

$\therefore \frac{dr}{dt} = (16t)i + (2t-4)j + K$

$A = \frac{d^2r}{dt^2} = 16i + 2j + K$

$|A| = \left|\frac{d^2r}{dt^2}\right|_{t=1} = \sqrt{16^2 + 2^2 + 1^2}$

$= \sqrt{256 + 4 + 1}$

$= \sqrt{261} = 16.155$ Units.

$|A| = 16.16$ Units.

(3) $A = 4i + 2j - 4K$, $B = 8i - 2j + K$, $C = i + 4j - 3K$; find the vector triple product $(A \times B) \times C$

$$A \times B = \begin{vmatrix} i & j & K \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix} = i\begin{bmatrix} 2 & -4 \\ -2 & 1 \end{bmatrix} - j\begin{bmatrix} 4 & -4 \\ 8 & 1 \end{bmatrix} + K\begin{bmatrix} 4 & 2 \\ 8 & -2 \end{bmatrix}$$

$= i[2-8] - j[4-(-32)] + K[-8-16]$

$A \times B = -6i + 36j - 24K$

$$(A \times B) \times C = \begin{vmatrix} i & j & K \\ -6 & 36 & -24 \\ 1 & 4 & -3 \end{vmatrix} = i\begin{bmatrix} 36 & -24 \\ 4 & -3 \end{bmatrix} - j\begin{bmatrix} -6 & -24 \\ 1 & -3 \end{bmatrix} + K\begin{bmatrix} -6 & 36 \\ 1 & 4 \end{bmatrix}$$

$= i[-108-(-96)] - j[18-(-24)] + K[-24-36]$

$(A \times B) \times C = -12i - 42j - 60K$