QUESTION B

For a pair of line to be perpendicular

$$m_1 m_2 = -1$$

1. $y - 3x - 2 = 0$ ... eq 1

$y - 3x = 2$

$y + mx = c$

$m_1 = -3$

$3y + x + 9 = 0$ ... eq 1

$3y + x = -9$

$y + \frac{1}{3}x = -\frac{9}{3}$

$y + \frac{1}{3}x = -3$

$y + mx = c$

$m_2 = \frac{1}{3}$

$$m_1 m_2 = -3 \times \frac{1}{3} = \underline{-1}$$

Therefore $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

2. $3y - 4 = 2x + 3$ ... eq 1

$3y - 2x = 3 + 4$

$3y - 2x = 7$

$y - \frac{2}{3}x = 7/3$

$y + mx = c$

$m_1 = -2/3$

And

$y - 5 = x + 6$

$y - x = 6 + 5$

$y - x = 11$

$y + x = 6$

$m_2 = -1$

$m_1 \times m_2 = -\frac{2}{3} \times -1 = \frac{2}{3}$

Therefore Lines $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular

3. $x^2 + y^2 + 3xy - 11 = 0$ at point $(1, 2)$

Differentiating

$2x + 2y\frac{dy}{dx} + 3\left[x\frac{dy}{dx} + y\right] + 0 = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y + 2x = 0$

$\frac{dy}{dx}\left[2y + 3x\right] + 3y + 2x = 0$

$\frac{dy}{dx} = \frac{-3y - 2x}{3x + 2y} = m$

at $x = 1$ and $y = 2$

$\frac{dy}{dx} = \frac{-3(2) - 2(1)}{3(1) + 2(2)}$

$\frac{dy}{dx} = m = \frac{-6 - 2}{3 + 4} = -\frac{8}{7}$

To find the tangent

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{8}{7}(x - 1)$

$2y - 14 = -8x + 8$

$2y + 8x - 14 - 8 = 0$

$2y + 8x - 22 = 0$

$2y + 8x = 22$

To the equation of the normal

$y - y_1 = -\frac{1}{m}(x - x_1)$

$y - 2 = -\frac{1}{-\frac{8}{7}}(x - 1)$

$y - 2 = \frac{7}{8}(x - 1)$

$8y - 16 = 7x - 7$

$8y - 7x = -7 + 16$

$8y - 7x = 9$