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Assignment for Dr. Ogelami's Group

(1) find the limit of the function $(4x^2 - \sin x)/x^3)$ as $x \to 0$

solution

Given $\frac{4x^2 - \sin x}{x^3}$, by direct substitution as x tends to 0, we have:

$$\frac{4(0) - \sin 0}{0^3} = \frac{0-0}{0} = \frac{0}{0} = \text{undefined.}$$

Using L'hopital's rule:

$$\lim_{x \to 0} \left(\frac{4x^2 - \sin x}{x^3} \right) = \frac{8x - \cos x}{3x^2} = \frac{8x - \cos x}{3x^2}$$

i.e by differentiating:

$\therefore \lim_{x \to 0} \left(\frac{8(0) - \cos(0)}{3(0)^2} \right) = \frac{0 - 1}{0} = \frac{-1}{0} =$ undefined

$\therefore$ by further differentiation:

$\frac{8x - \cos x}{3x^2} = \frac{8 - (-\sin x)}{6x} = \frac{8 + \sin x}{6x}$

$\therefore \lim_{x \to 0} \left(\frac{8 + \sin x}{6x} \right) = \frac{8 + \sin 0}{6(0)} =$ undefined

by further differentiation,

$\frac{8 + \sin x}{6x} = \frac{\cos x}{6}$

$\lim_{x \to 0} = \left(\frac{\cos 0}{6} \right) = 1/6$ ans'

EV1

2) If $y = (9x^2 \cos 8x)/e^{3x}$, find the derivative of y with respect to x.

Solution

Given $y = \dfrac{9x^2 \cos 8x}{e^{3x}}$

let $u = 9x^2$, $\dfrac{du}{dx} = 18x$

$v = \cos 8x$, $dv/dx = -8 \sin 8x$

$w = e^{3x}$ $\dfrac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left(\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \times \frac{dv}{dx} - \frac{1}{w} \times \frac{dw}{dx}\right)$$

$$\therefore \frac{dy}{dx} = y\left(\frac{1}{\cancel{9x^2}} \times \cancel{18x} + \frac{1}{\cos 8x} \times -8\sin 8x - \frac{1}{\cancel{e^{3x}}} \times 3\cancel{e^{3x}}\right)$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{8\sin 8x}{\cos 8x} - 3\right)$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 8\tan 8x - 3\right) \text{ ans.}$$

Partner Évolution

(3) If $y = \cos(5x^2+6x)$ find dy/dx

solution

Given $y = \cos(5x^2+6x)$

let $u = 5x^2+6x$

$\therefore \frac{du}{dx} = 10x+6$

$\therefore y = \cos u$

$\frac{dy}{du} = -\sin u.$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = 10x+6 \times -\sin u$

$\frac{dy}{dx} = -\sin u \times 10x+6$

$\frac{dy}{dx} = -\sin u(10x+6)$, but $u = 5x^2+6x$

$\therefore \frac{dy}{dx} = -\sin 5x^2+6x(10x+6)$ ans.

4) find the Integral of the following

a) $\frac{3}{(4x+1)} dx$

Solution

$$\int \frac{3}{4x+1} \, dx$$

let $u = 4x+1$, $\frac{du}{dx} = \frac{4}{1}$

$\therefore dx = \frac{du}{4}$

but $x = \frac{u-1}{4}$

$$\int \frac{du}{4} \times 3 \times \frac{1}{4x+1} = \int \frac{3\,du}{4(4x+1)}$$

$$\therefore \frac{3}{4}\int \frac{du}{4x+1} = \frac{3}{4}\int \frac{du}{u} = \frac{3}{4} \times \frac{2}{u} + C$$

by final simplification,

$$\frac{3}{2(4x+1)^2} + C$$

ans.

b) $\dfrac{dx}{(x^2+49)}$

Solution

$$\int \frac{dx}{(x^2+49)} = \int \frac{dx}{x^2+7^2}$$

$$x = 7\tan\theta$$

$$\frac{dx}{d\theta} = 7\sec^2\theta, \quad dx = 7\sec^2\theta\, d\theta$$

$$x^2+7^2 = 7\tan^2\theta + 7^2 = 7^2(\tan^2\theta+1)$$

factorizing after substituting

$$\int \frac{\cancel{7}\,\cancel{\sec^2\theta}\, d\theta}{\cancel{49}^{\,7}\,\cancel{\sec^2\theta}} = \int \frac{d\theta}{7} = \frac{1}{7}\int d\theta = \frac{1}{7}\theta + C$$

but $\theta = \tan^{-1}\dfrac{x}{7}$

$$= \tfrac{1}{7}\tan^{-1}\frac{x}{7} + C$$

ans

(c) $\left(e^{6x}+9x^3-\sin 7x+\cos 8x\right)dx$

solution

$\int e^{6x}+9x-\sin 7x+\cos 8x\,dx$

$= \frac{1}{6}e^{6x}+\frac{9x^2}{2}+\frac{1}{7}\cos 7x+\frac{1}{8}\sin 8x+c$

ans.

d) $x\sqrt{9+x^2}\,dx$

solution

$\int x\sqrt{9+x^2}\,dx$

let $u=9+x^2$, $x=\sqrt{u-9}$

$\frac{du}{dx}=2x \therefore dx=\frac{du}{2x}$

from $\int x\sqrt{9+x^2}\,dx$, substituting

$\int x\,u^{1/2}\,\frac{du}{2x}$

$\int(\sqrt{u-9})\,u^{1/2}\,\frac{du}{2x} = \int \cancel{(u-9)}^{1/2}\,u^{1/2}\,\frac{du}{2(u-9)^{1/2}}$

$=\frac{1}{2}\int u^{1/2}\,du$, integrating;

$=\frac{1}{2}\left(\frac{u^{3/2}}{1}\times\frac{2}{3}\right)=\frac{1}{2}\times\frac{2u^{3/2}}{3}=\frac{1}{3}u^{3/2}$ ans.