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DEPARTMENT: MECHATRONICS ENGINEERING
COURSE: MAT 104

COVID – 19 HOLIDAY ASSIGNMENT

QUESTION 1

Find the derivatives using the first principle

(a) $y = Sin(\frac{3}{x^2})$

(b) $y = \frac{4}{x^3}$

SOLUTION

(a) $y = Sin(\frac{3}{x^2})$

$f_{(x)} = Sin(\frac{3}{x^2})$

$f_{(x + \partial x)} = Sin(\frac{3}{(x + \partial x)^2})$

$\frac{\partial y}{\partial x} = \frac{f_{(x + \partial x)} - f_{(x)}}{\partial x}$

$\frac{\partial y}{\partial x} = \frac{Sin\left(\frac{3}{(x + \partial x)^2}\right) - Sin(\frac{3}{x^2})}{\partial x}$

$\frac{\partial y}{\partial x} = \frac{Sin\left(\frac{3}{(x^2 + 2x\partial x + \partial^2 x)}\right) - Sin(\frac{3}{x^2})}{\partial x}$

$= \frac{1}{\partial x}[Sin\left(\frac{3}{(x^2 + 2x\partial x + \partial^2 x)} - \frac{3}{x^2}\right)]$

$= \frac{1}{\partial x}[Sin\left(\frac{3x^2 - 3(x^2 + 2x\partial x + \partial^2 x)}{x^2(x^2 + 2x\partial x + \partial^2 x)}\right)]$

$= \frac{1}{\partial x}[Sin\left(\frac{3x^2 - 3x^2 - 6x\partial x - 3\partial^2 x}{x^2(x^2 + 2x\partial x + \partial^2 x)}\right)]$

$= \frac{1}{\partial x}Sin . \frac{1}{\partial x}\left(\frac{- 6x\partial x - 3\partial^2 x}{x^2(x^2 + 2x\partial x + \partial^2 x)}\right)$

$= Cos(\frac{3}{x^2}) \times \frac{1}{\partial x}\left(\frac{\partial x(- 6x - 3\partial x)}{x^2(x^2 + 2x\partial x + \partial^2 x)}\right)$

$= Cos(\frac{3}{x^2}) \times \left(\frac{- 6x - 3\partial x}{x^2(x^2 + 2x\partial x + \partial^2 x)}\right)$

As $\partial x \to 0$

$= Cos(\frac{3}{x^2}) \times \left(\frac{- 6x}{x^4}\right)$

$= Cos(\frac{3}{x^2}) \times \left(\frac{- 6}{x^3}\right)$

$\therefore \frac{\partial y}{\partial x} = \frac{-6Cos(\frac{3}{x^2})}{x^3}$

(b) $y = \frac{4}{x^3}$

$$f_{(x)} = \frac{4}{x^3}$$

$$f_{(x+\partial x)} = \frac{4}{(x+\partial x)^3} = \frac{4}{x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x}$$

$$\frac{\partial y}{\partial x} = \frac{f_{(x+\partial x)} - f_{(x)}}{\partial x}$$

$$= \frac{\frac{4}{x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x} - \frac{4}{x^3}}{\partial x}$$

$$= \frac{1}{\partial x} \times \left(\frac{4}{x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x} - \frac{4}{x^3}\right)$$

$$= \frac{1}{\partial x}\left(\frac{4x^3 - 4(x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x)}{x^3(x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x)}\right)$$

$$= \frac{1}{\partial x}\left(\frac{4x^3 - 4x^3 - 12x^2\partial x - 12x\partial^2 x - 4\partial^3 x)}{x^3(x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x)}\right)$$

$$= \frac{1}{\partial x}\left(\frac{-12x^2\partial x - 12x\partial^2 x - 4\partial^3 x}{x^3(x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x)}\right)$$

$$= \frac{1}{\partial x}\left(\frac{\partial x(-12x^2 - 12x\partial x - 4\partial^2 x)}{x^3(x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x)}\right)$$

$$= \left(\frac{-12x^2 - 12x\partial x - 4\partial^2 x}{x^3(x^3 + 3x^2\partial x + 3x\partial^2 x + \partial^3 x)}\right)$$

As $\partial x \to 0$

$$= \frac{-12x^2}{x^6}$$

$$\therefore \frac{\partial y}{\partial x} = \frac{-12}{x^4}$$

QUESTION 2

Find the integral of the following

(a) $\frac{\partial x}{(x^2 + 36)}$

(b) $\frac{\partial x}{(x^2 + 13)}$

SOLUTION

(a) $\frac{\partial x}{(x^2 + 36)}$

$$\frac{\partial x}{(x^2 + 36)} = \int \frac{1}{x^2 + 36}\partial x$$

Substituting $u = \frac{x}{6}$

This implies that $x = 6u$

But $\frac{\partial u}{\partial x} = \frac{1}{6}$

This implies that $\partial x = 6\partial u$

$$\int \frac{1}{x^2 + 36}\partial x = \int \frac{6}{(6u)^2 + 36}\partial u = \int \frac{6}{36u^2 + 36}\partial u$$

$$= \frac{1}{6}\int \frac{1}{u^2 + 1}\partial u$$

$$= \int \frac{1}{u^2 + 1}\partial u = \tan^{-1}(u) + C$$

$$= \frac{1}{6}\,.\tan^{-1}(u) + C$$
$$= \frac{\tan^{-1}(\frac{x}{6})}{6} + C$$

(b) $\frac{\partial x}{(x^2 + 13)}$

$\frac{\partial x}{(x^2 + 13)} = \int \frac{1}{x^2 + 13}\partial x$

Substituting $u = \frac{x}{\sqrt{13}}$

This implies that $x = \sqrt{13}u$

But $\frac{\partial u}{\partial x} = \frac{1}{\sqrt{13}}$

This implies that $\partial x = \sqrt{13}\partial u$

$$\int \frac{1}{x^2 + 13}\partial x = \int \frac{\sqrt{13}}{(\sqrt{13}u)^2 + 13}\partial u = \int \frac{\sqrt{13}}{13u^2 + 13}\partial u$$
$$= \frac{\sqrt{13}}{13}\int \frac{1}{u^2 + 1}\partial u$$
$$= \frac{1}{\sqrt{13}}\int \frac{1}{u^2 + 1}\partial u$$
$$= \int \frac{1}{u^2 + 1}\partial u = \tan^{-1}(u) + C$$
$$= \frac{1}{\sqrt{13}}\,.\tan^{-1}(u) + C$$
$$= \frac{\tan^{-1}(\frac{x}{\sqrt{13}})}{\sqrt{13}} + C$$