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COVID – 19 HOLIDAY ASSIGNMENT

QUESTIONS

Find the integral of the following:

(a) $x^2 \sin x \, \partial x$
(b) $3te^{2t} \partial t$
(c) $2x^2 \ln x \, \partial x$
(d) $\frac{(2x - 3x^2)}{(1 - x)} \partial x$

SOLUTION

(a) $x^2 \sin x \, \partial x$

$\int x^2 \sin x \, \partial x$

Using integration by parts

$\int \mathrm{U} \, \partial \mathrm{v} = \mathrm{UV} - \int \mathrm{V} \, \partial \mathrm{u}$

U = x^2

∂v = sin$x$

$\int \partial$v = V = $-\cos x$

$\frac{\partial u}{\partial x} = 2x$

$\int x^2 \sin x \, \partial x = x^2 . -\cos x - \int -\cos x \, . \, 2x \, \partial x$

$= -x^2 \cos x - \int -2x \cos x \, \partial x$

But $\int -2x \cos x \, \partial x$

$= -2 \int x \cos x \, \partial x$

Integrating $\int x \cos x \, \partial x$ by parts

$\int x \cos x \, \partial x = x \sin x - \int \sin x \, \partial x$

$= x \sin x - (-\cos x)$

$= x \sin x + \cos x$

Thus $-2 \int x \cos x \, \partial x = -2(x \sin x + \cos x)$

$= -2x \sin x - 2\cos x$ + C

Hence $\int x^2 \sin x\, \partial x = -x^2 \cos x - (-2x\sin x - 2\cos x)$

$= -x^2 \cos x + 2x\sin x + 2\cos x + C$

$= 2x\sin x - x^2 \cos x + 2\cos x + C$

$\therefore \int x^2 \sin x\, \partial x = 2x\sin x + (2 - x^2)\cos x + C$

(b) $3te^{2t}\partial t$

$\int 3te^{2t}\partial t$
$= 3\int te^{2t}\partial t$

Using integration by parts

$\int \mathrm{U}\, \partial \mathrm{v} = \mathrm{UV} - \int \mathrm{V}\, \partial \mathrm{u}$

U = t

∂v = e^{2t}

$\int \partial$v = V = $\frac{e^{2t}}{2}$

$\frac{\partial u}{\partial t} = 1$

$\int te^{2t}\partial t = t \,.\, \frac{e^{2t}}{2} - \int \frac{e^{2t}}{2} .1\,\partial t$

$= \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\partial t$

But $\int \frac{e^{2t}}{2}\partial t$

Substituting $u = 2t$

This implies that $\frac{\partial u}{\partial t} = 2$

Thus $\partial t = \frac{1}{2}\partial u$

$= \frac{1}{2}\int e^{u} . \frac{1}{2}\partial u$

$= \frac{1}{4}\int e^{u}\,\partial u$

$= \frac{1}{4} . e^{u}$

$= \frac{e^{u}}{4}$

$\int te^{2t}\partial t = \frac{te^{2t}}{2} - \int \frac{e^{2t}}{2}\partial t$

$$= \frac{te^{2t}}{2} - \frac{e^{u}}{4}$$

$$= \frac{te^{2t}}{2} - \frac{e^{2t}}{4}$$

Thus $3\int te^{2t}\partial t = 3(\frac{te^{2t}}{2} - \frac{e^{2t}}{4})$

$$= \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$$

Hence $\int 3te^{2t}\partial t = \frac{3te^{2t}}{2} - \frac{3e^{2t}}{4} + C$

$\therefore \int 3te^{2t}\partial t = \frac{3(2t - 1)e^{2t}}{4} + C$

(c) $2x^2 \ln x \, \partial x$

$\int 2x^2 \ln x \, \partial x$

$= 2\int x^2 \ln x \, \partial x$

Using integration by parts

$\int U \, \partial v = UV - \int V \, \partial u$

U = $\ln x$

∂v = x^2

$\int \partial v = V = \frac{x^3}{3}$

$\frac{\partial u}{\partial x} = \frac{1}{x}$

$\int x^2 \ln x \, \partial x = \ln x \, . \, \frac{x^3}{3} - \int \frac{x^3}{3} . \frac{1}{x} \partial x$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} \partial x$$

But $\int \frac{x^2}{3} \partial x$

$$= \frac{1}{3} \int x^2 \, \partial x$$

Also $\int x^2 \, \partial x = \frac{x^3}{3}$

And $\frac{1}{3} \int x^2 \, \partial x = \frac{1}{3} \times \frac{x^3}{3} = \frac{x^3}{9}$

Thus $\frac{x^3 \ln x}{3} - \int \frac{x^2}{3} \partial x$

$= \frac{x^3 \ln x}{3} - \frac{x^3}{9}$

Hence, $2\int x^2 \ln x \, \partial x = 2(\frac{x^3 \ln x}{3} - \frac{x^3}{9})$

$= \frac{2x^3 \ln x}{3} - \frac{2x^3}{9} + C$

$\therefore \int 2x^2 \ln x \, \partial x = \frac{2x^3(3 \ln x - 1)}{9} + C$

(d) $\frac{(2x - 3x^2)}{(1 - x)} \partial x$

$\int \frac{(2x - 3x^2)}{(1 - x)} \partial x$

$= \int \frac{x(3x - 2)}{(x - 1)} \partial x$

Substituting $u = x - 1$

This implies that $\frac{\partial u}{\partial x} = 1$

Thus $\partial x = \partial u$

$= \int \frac{(u + 1)(3u + 1)}{u} \partial u$

$= \int \frac{3u^2 + u + 3u + 1}{u} \partial u$

$= \int \frac{3u^2 + 4u + 1}{u} \partial u$

$= \int (3u + 4 + \frac{1}{u}) \partial u$

$= \int 3u \partial u + \int 4 \partial u + \int \frac{1}{u} \partial u$

$= 3\int \mathrm{U} \partial u + 4\int 1 \partial u + \int \frac{1}{u} \partial u$

$= 3 . \frac{u^2}{2} + 4 . u + \ln u + C$

$= \frac{3u^2}{2} + 4u + \ln u + C$

$= \frac{3(x - 1)^2}{2} + 4(x - 1) + \ln(x - 1) + C$

$\therefore \int \frac{(2x - 3x^2)}{(1 - x)} \partial x = 4(x - 1) + \frac{3(x - 1)^2}{2} + \ln(x - 1) + C$