Collins Sarima Lennenda

19/sci01/034

Collins Sarima Lennenda
19/sci01/034
Computer science

1) Find the limit of the function $(4x^2 - \sin x / x^3)$ as $x \to 0$.

solution

$$\lim_{x \to 0} \left\{ \frac{4x^2 - \sin x}{x^3} \right\}$$

By l'hopital's rule we have

$$\lim_{x \to 0} \left\{ \frac{8x - \cos x}{3x^2} \right\}$$

for the second derivative we have

$$\lim_{x \to 0} \left\{ \frac{8 + \sin x}{6x} \right\}$$

for the third derivative

$$\lim_{x \to 0} \left\{ \frac{\cos x}{6} \right\}$$

$$= \frac{\cos(0)}{6} = \frac{1}{6}$$

2 If $y = (7x\cos 8x)/e^{3x}$, find the ~~derivate~~ derivative of y with respect to x.

solution

$$y = \frac{7x^2 \cos 8x}{e^{3x}}$$

$u = 7x^2$ $\quad v = \cos 8x$ $\quad w = e^{3x}$

$\frac{du}{dx} = 14x$ $\quad \frac{dv}{dx} = -8\sin 8x$ $\quad \frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left[\frac{1}{u}\cdot\frac{du}{dx} + \frac{1}{v}\cdot\frac{dv}{dx} - \frac{1}{w}\cdot\frac{dw}{dx}\right]$$

$$= \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{1}{7x^2}(14x) + \frac{1}{\cos 8x}(-8\sin 8x) - \frac{1}{e^{3x}}(3e^{3x})\right.$$

$$\frac{dy}{dx} = \frac{7x^2\cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x - 3\right]$$

3 If $y = \cos(5x^2 + 6x)$. Find dy/dx

solution

$y = \cos(5x^2 + 6x)$

$u = 5x^2 + 6x$ $\quad dy/dx = 10x + 6$

$y = \cos u$ $\quad dy/du = -\sin u$

$dy/dx = dy/dx \times dy/du$

$= (10x + 6) \times -\sin u$

$= -(10x+6)\sin u$ $\quad$ Recall $u = (5x^2 + 6x)$

$= -(10x+6)\sin(5x^2 + 6x)$

4 find the integral of the following

a) $\int \frac{3}{4x+1}dx = 3\int \frac{dx}{4x+1}$

$u = 4x + 1$

$dy/dx = 4$

$dx = \cancel{4du}\ \frac{du}{4}$

$$3\int\frac{dx}{4x+1} = 3\int\frac{1}{u}\cdot\frac{du}{4} = \frac{3}{4}\int\frac{1}{u}du$$

$= \frac{3}{4}\ln u + C$ $\quad$ Recall $u = 4x + 1$

$= \frac{3\ln(4x+1)}{4} + C$

b $\int \frac{dx}{x^2+49} = \int\frac{1}{x^2+49}dx = \int\frac{1}{49(\frac{x^2}{49}+1)}dx$

$\frac{1}{49}\int\frac{1}{(\frac{x}{7})^2 + 1}dx$ $\quad u = x/7$, $du = 1/7\,dx$

$7du = dx$

$\frac{1}{49}\int\frac{1}{u^2+1}7du$

$$\frac{1}{49} \cdot \frac{7}{1} \int \frac{1}{u^2+1} \, du$$

$$\frac{1}{7} \int \frac{1}{u^2+1} \, du$$

$$= \frac{1}{7} \tan^{-1} \frac{x}{7} + C$$

c) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x) \, dx$

$$\frac{1}{6} e^{6x} + \frac{9x^{3+1}}{3+1} - \left(-\frac{1}{7} \cos 7x\right) + \frac{1}{8} \sin 8x + C$$

$$\frac{1}{6} e^{6x} + \frac{9x^4}{4} + \frac{1}{7} \cos 7x + \frac{1}{8} \sin 8x + C$$

d $\int x \sqrt{9+x^2} \, dx$

$u = 9 + x^2 \quad du/dx = 2x \, , \quad dx = \frac{du}{2x}$

$$\int \cancel{x} \sqrt{u} \, \frac{du}{2\cancel{x}}$$

$$= \frac{1}{2} \int u^{1/2} \, du$$

$$= \frac{1}{2} \left[\frac{u^{3/2}}{3/2} \right] + C$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} \left[u^{3/2} \right] + C$$

$$= \frac{1}{3} \left[u^{3/2} \right] + C$$

Recall $u = 9 + x^2$

$$= \frac{1}{3} \left[(9+x^2)^{3/2} \right] + C$$

$$= \frac{(9+x^2)^{3/2}}{3} + C$$