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MATRIC NO: 19/ENG09/016 GENERAL MATHEMATICS III

DEPT: AERONAUTICAL ENGINEERING. MAT 104.

ASSIGNMENT FOR DR. OYELAMIS GROUP

i) Find the limit of the function $\left((4x^2 - \sin x)/x^3\right)$ as $x \to 0$.

Solution.

Given $\frac{4x^2 - \sin x}{x^3}$, by direct substitution as x tends to 0, we have:

$$\frac{4(0) - \sin 0}{0^3}$$

$$= \frac{0-0}{0} = \frac{0}{0} = \text{undefined}$$

Use L'Hopital's rule.

$$\lim_{x \to 0} \left(\frac{4x^2 - \sin x}{x^3}\right) = \frac{8x - \cos x}{3x^2} = \frac{8x - \cos x}{3x^2}.$$

i.e by differentiating.

$$\therefore \lim_{x \to 0} \left(\frac{8(0) - \cos 0}{3(0)^2}\right) = \frac{0-1}{0} = \frac{-1}{0} = \text{Undefined}$$

$\therefore$ by further differentiation:

$$\frac{8x - \cos x}{3x^2} = \frac{8 - (-\sin x)}{6x} = \frac{8 + \sin x}{6x}.$$

$$\therefore \lim_{x \to 0} \left(\frac{8 + \sin x}{6x}\right) = \frac{8 + \sin 0}{6(0)} = \text{Undefined.}$$

By further differentiation,

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$$\frac{8 + \sin x}{6x} = \frac{\cos x}{6}.$$

$$\lim_{x \to 0} = \left(\frac{\cos 0}{6}\right) = \frac{1}{6} \,//$$

2) If $y = (7x^2 \cos 8x) / e^{3x}$, find the derivative of y with respect to x.

Solution

Given $y = \dfrac{7x^2 \cos 8x}{e^{3x}}$.

let $u = 7x^2$, $\dfrac{du}{dx} = 14x$,

$v = \cos 8x$, $\dfrac{dv}{dx} = -8\sin 8x$

$w = e^{3x}$ $\dfrac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y\left(\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \times \frac{du}{dx} - \frac{1}{w} \times \frac{dw}{dx}\right)$$

$$\frac{dy}{dx} = y\left(\frac{1}{7x^2} \times 14x + \frac{1}{\cos 8x} \times -8\sin 8x - \frac{1}{e^{3x}} \times 3e^{3x}\right)$$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{8\sin 8x}{\cos 8x} - 3\right)$$

$$dy/dx = y\left(2/x - 8\tan 8x - 3\right) \,//$$

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3) 4 $y = \cos(5x^2+6x)$. find dy/dx DATE

Solution.

Given $y = \cos(5x^2 + 6x)$.

let $u = 5x^2 + 6x$.

$\therefore \frac{du}{dx} = 10x + 6$

$\therefore y = \cos u$.

$\frac{dy}{du} = -\sin u$.

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} = 10x + 6 \times -\sin u$.

$\frac{dy}{dx} = -\sin u \times 10x + 6$.

$\frac{dy}{dx} = -\sin u\,(10x + 6)$, but $u = 5x^2+6x$

$\therefore \frac{dy}{dx} = -\sin 5x^2 + 6x\,(10x + 6)$ //

4). Find the Integral of the following:

a) $\frac{3}{(4x+1)}\,dx$

Solution.

$\int \frac{3}{4x+1}\,dx$, let $u = 4x+1$, $\frac{du}{dx} = \frac{4}{1}$

$\therefore dx = \frac{du}{4}$. but $x = \frac{u-1}{4}$

$\int \frac{du}{4} \times 3 \times \frac{1}{4x+1} = \int \frac{3\,du}{4(4x+1)}$

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Integrating,

$= \frac{3}{4}\int\frac{du}{4x+1} = \frac{3}{4}\int\frac{du}{u} = \frac{3}{4_2}\times\frac{u^{1}}{u^2} + c$

by final simplification, $= \frac{3}{2u^2} + C$

$= \frac{3}{2(4x+1)^2} + C$

b) $\frac{dx}{(x^2+49)}$

Solution.

$\int\frac{dx}{(x^2+49)} = \int\frac{dx}{x^2+7^2}$

$x = 7\tan\theta$.

$\frac{dx}{d\theta} = 7\sec^2\theta$, $dx = 7\sec^2\theta\,d\theta$

$x^2+7^2 = 7\tan^2\theta + 7^2 = 7^2(\tan^2\theta+1)$.

factorizing after substituting.

$\int\frac{\cancel{7}\,\cancel{\sec^2}\theta\,d\theta}{_{7}\cancel{49}\,\cancel{\sec^2\theta}} = \int\frac{d\theta}{7} = \frac{1}{7}\int d\theta = \frac{1}{7}\theta + c$.

but $\theta = \tan^{-1}\frac{x}{7}$.

$= \frac{1}{7}\tan^{-1}\frac{x}{7} + C$

c) $(e^{6x} + 9x^3 - \sin 7x + \cos 8x)\,dx$.

Solution:

$\int e^{6x} + 9x - \sin 7x + \cos 8x\, dx$

$= \frac{1}{6}e^{6x} + \frac{9x^2}{2} + \frac{1}{7}\cos 7x + \frac{1}{8}\sin 8x + C$

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d) $x\sqrt{9+x^2}\,dx$

Solution

$$\int x\sqrt{9+x^2}\,dx.$$

let $u = 9+x^2$, $x = \sqrt{u-9}$

$\frac{dy}{dx} = 2x$ ∴ $dx = \frac{du}{2x}$

from $\int x\sqrt{9+x^2}\,dx$, substituting,

$$\int x\,u^{1/2}\,\frac{du}{2x}$$

$$\int \left(\sqrt{u-9}\right)u^{1/2}\,\frac{du}{2x} = \int \cancel{(u-9)^{1/2}}\,u^{1/2}\,\frac{du}{2\cancel{\left((u-9)^{1/2}\right)}}$$

$$= \frac{1}{2}\int u^{1/2}\,du.$$

Integrating

$$= \frac{1}{2}\left(\frac{u^{1/2+1}}{1/2+1}\right) = \frac{1}{2}\,\frac{u^{\frac{3}{2}}}{3/2}$$

$$= \frac{1}{2}\left(\frac{u^{3/2}}{1}\times\frac{2}{3}\right) = \frac{1}{\cancel{2}}\times\frac{\cancel{2}u^{3/2}}{3} = \frac{u^{3/2}}{3} = \frac{1}{3}u^{3/2}\ //$$