

$$1. \quad x^{\frac{1}{2}} \ln x$$

$$u = x^{\frac{1}{2}} \quad dv = \ln x$$

$$\frac{du}{dx} = \frac{1}{2} x^{-1/2}$$

$$\int \frac{v du}{dx} + \int \frac{u dv}{dx}$$

$$\ln x \int x^{\frac{1}{2}} dx + x^{\frac{1}{2}} \int \frac{d \ln x}{dx}$$

$$\ln x \left[ \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right] + x^{\frac{1}{2}} \left[ \frac{1}{x} \right] + C$$

$$\frac{2 \ln x}{3} \cdot x^{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{x} + C$$

$$\frac{2 x^{\frac{3}{2}} \ln x}{3} + \frac{\sqrt{x}}{x} + C$$

$$2. \quad \int 2 \cos bt \cos t dt = 2 \int \cos bt \cos t dt$$

$$A = bt \quad B = t$$

$$\begin{aligned} \cos A \cos B &= \frac{1}{2} [\cos(A+B) + \cos(A-B)] \\ &= \frac{1}{2} [\cos(bt+t) + \cos(bt-t)] \\ &= \frac{1}{2} [\cos 7t + \cos 5t] \end{aligned}$$

$$\begin{aligned} \int 2 \cos bt \cos t dt &= \frac{1}{2} \int (2 \cos 7t + 2 \cos 5t) dt \\ &= \frac{2}{2} \left[ \frac{\sin 7t}{7} - \frac{\sin 5t}{5} \right] \end{aligned}$$

$$= \frac{\sin 7t}{7} - \frac{\sin 5t}{5} + C$$

$$3. \int \sin^3 x \cos^4 x dx$$

$$u = \cos x$$

$$\frac{du}{dx} = -\sin x \Rightarrow dx = \frac{du}{-\sin x}$$

$$\int \cos^4 x \sin^3 x dx$$

$$\int u^4 \sin^2 x du \rightarrow \int u^4 \sin^2 x - \frac{du}{\sin x}$$

$$= \int u^4 \sin^2 x - du$$

$$- \int u^4 \sin^2 x \cdot du$$

Recall that

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$- \int \sin^2 x u^4 \cdot du$$

$$= \int (1 - \cos^2 x) u^4 du$$

$$\text{but } (1 - \cos^2 x)$$

$$\text{but } u = \cos x$$

$$- \int (1 - u^2) u^4 du$$

$$= \int (u^4 - u^6) du$$

$$= \left[ \frac{u^{4+1}}{4+1} - \frac{u^{6+1}}{6+1} \right] + C$$

$$= \left[ \frac{u^5}{5} - \frac{u^7}{7} \right] + C$$

$$\frac{(\cos x)^5}{5} - \frac{(\cos x)^7}{7} + C$$