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DEPAERTMENT: COMPUTER SCIENCE

MATRIC NUMBER: 19/SCI01/015

ASSIGNMENT

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1. $\lim_{x \to 0} \frac{(4x^2 - \sin x)}{x^3}$

solution

Using L'hopital rule, we have

$= \lim_{x \to 0} = \frac{(8x - (\cos x))}{3x^2}$

$= \lim_{x \to 0} \frac{(8 - (-\sin x))}{6x} = \lim_{x \to 0} \frac{(8 + \sin x)}{6x}$

$= \lim_{x \to 0} \frac{(\cos x)}{6} = \frac{\cos(0)}{6} = \frac{1}{6}$

2. $y = \frac{7x^2 \cos 8x}{e^{3x}}$

Soln

$u = 7x^2, \quad v = \cos 8x, \quad w = e^{3x}$

$\frac{du}{dx} = 14x, \quad \frac{dv}{dx} = -8\sin 8x$ and $\frac{dw}{dx} = 3e^{3x}$

$\frac{dy}{dx} = y\left[\frac{1}{u}\frac{du}{dx} + \frac{1}{v}\frac{dv}{dx} - \frac{1}{w}\frac{dw}{dx}\right]$

$\frac{dy}{dx} = y\left[\frac{1}{7x^2}(14x) + \frac{1}{\cos 8x}(-8\sin 8x) - \frac{1}{e^{3x}}(3e^{3x})\right]$

$\frac{dy}{dx} = y\left[\frac{\cancel{14x}}{\cancel{7x^2}} + \frac{(-8\sin 8x)}{\cos 8x} - \frac{3\cancel{e^{3x}}}{\cancel{e^{3x}}}\right]$

$\frac{dy}{dx} = y\left[\frac{2}{x} - 8\tan 8x - 3\right]$

$\frac{dy}{dx} = \frac{7x^2 \cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x - 3\right]$

$\frac{dy}{dx} = \frac{14\cancel{x}^2 \cos 8x}{\cancel{x} e^{3x}} - \frac{56x^2 \tan 8x \cos 8x}{e^{3x}} - \frac{21x^2 \cos 8x}{e^{3x}}$

$$\frac{dy}{dx} = \frac{14x\cos 8x - 56x^2\sin 8x - 21x^2\cos 8x}{e^{3x}}$$

3. $y = \cos(5x^2+6x)$

soln

using chain rule

$u = 5x^2+6x$

$\frac{du}{dx} = 10x+6$

$\frac{dy}{du} = \cos u$

$= -\sin u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\sin u \times (10x+6)$$

$$= -\sin(5x^2+6x) \times (10x+6)$$

$$= -\sin(5x^2+6x)(10x+6)$$

4.

a. $\int \frac{3\,dx}{(4x+1)}$

Solution

$\int \frac{3\,dx}{(4x+1)}$

$u = 4x+1$

$\frac{du}{dx} = 4$

$\frac{du}{4} = \frac{4\,dx}{4}$

$dx = \frac{1}{4}\,du$

$$= \int \frac{3}{(4x+1)}\,dx = \int \frac{3}{u} \cdot \frac{1}{4}\,du = \int \frac{3}{4u}\,du = \frac{3}{4}\int \frac{1}{u}\,du$$

$$= \frac{3}{4}\ln(u) + c$$

$= \frac{3}{4} \ln(4x+1) + c$

4b. $\int \frac{dx}{(x^2+49)}$

Soln

$\int \frac{dx}{(x^2+49)} = \int \frac{dx}{49\left(\frac{x^2}{49}+1\right)} = \int \frac{dx}{49\left(\frac{x}{7}\right)^2+1} = \frac{1}{49}\int \frac{dx}{\left(\frac{x}{7}\right)^2+1}$

$u = \frac{x}{7}$

$\frac{du}{dx} = \frac{1}{7}$

$dx = 7du$

$= \frac{1}{49}\int \frac{dx}{u^2+1} = \frac{1}{49}\int \frac{1}{u^2+1} \cdot 7du = \frac{\cancel{7}^{1}}{\cancel{49}_{7}}\int \frac{1}{u^2+1} du$

Recall, $\int \frac{1}{u^2+1} du$ is a standard integral, so it equates to $\arctan u$

$= \frac{1}{7} \times \arctan u + c$

$= \frac{\arctan(x/7)}{7} + c$

4c. $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\, dx$

Soln

$= \int e^{6x} dx + \int 9x^3 dx - \int \sin 7x\, dx + \int \cos 8x\, dx$

$= \frac{e^{6x}}{6} + \left[\frac{9x^4}{4}\right] - \frac{1}{7}\left(-\cos 7x\right) + \frac{1}{8}\left(\sin 8x\right) + C$

$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos 7x}{7} + \frac{\sin 8x}{8} + C$

4d. $\int x\sqrt{9+x^2}\,dx$

Soln

$\int x\sqrt{(9+x^2)}\,dx$

$u = 9+x^2$

$\frac{du}{dx} = 2x$

$\frac{du}{2x} = \frac{2\cancel{x}\,dx}{2\cancel{x}}$

$dx = \frac{du}{2x}$

$= \int x\sqrt{u}\cdot\frac{du}{2x}$

$= \int \frac{\cancel{x}\sqrt{u}}{2\cancel{x}}\,du$

$= \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\left[\frac{u^{1/2+1}}{1/2+1}\right] + c$

$= \frac{1}{2}\,\frac{u^{3/2}}{3/2}$

$= \frac{1}{2}\,u^{3/2} \times \frac{2}{3}$

$= \frac{\cancel{2}^{1}\,u^{3/2}}{\cancel{6}\,3}$

$= \frac{1}{3}\,u^{3/2} = \frac{1}{3}\left[9+x^2\right]^{3/2}$

$= \frac{(9+x^2)^{3\times 1/2}}{3}$

$= \frac{(9+x^2)(\sqrt{9+x^2})^2}{3} + c$