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1) $A = 3i + 7j - 2k$, $B = i + 3j + 7k$, $C = 9i - 4j + 6k$

a) The angle between A and C

Solution: $\cos \theta = \frac{\bar{A} \cdot \bar{C}}{|\bar{A}| |\bar{C}|}$

$$\bar{A} \cdot \bar{C} = (3i + 7j - 2k) \cdot (9i - 4j + 6k)$$
$$= 27 - 28 - 12$$

$$|\bar{A}| = \sqrt{3^2 + 7^2 - 2^2} = \sqrt{62}$$

$$|\bar{C}| = \sqrt{9^2 - 4^2 + 6^2} = \sqrt{133}$$

$$\cos \theta = \frac{27 - 28 - 12}{\sqrt{62} \sqrt{133}} = \frac{-13}{\sqrt{62} \sqrt{133}}$$

$$\theta = \cos^{-1}(-0.148)$$
$$= 98.2^\circ$$

b) The angle between B and C

Solution: $\cos \theta = \frac{\bar{B} \cdot \bar{C}}{|\bar{B}| |\bar{C}|}$

$$\bar{B} \cdot \bar{C} = (i + 3j + 7k) \cdot (9i - 4j + 6k)$$
$$= 9 - 12 + 42 = 39$$

$$|\bar{B}| = \sqrt{1^2 + 3^2 + 7^2} = \sqrt{59}$$

$$|\bar{C}| = \sqrt{9^2 - 4^2 + 6^2} = \sqrt{133}$$

$$\cos \theta = \frac{39}{\sqrt{59} \sqrt{133}} = 0.440$$

$$\theta = \cos^{-1}(0.440)$$
$$= 63.9^\circ$$

c) The unit vector in the direction of $(\bar{A} + \bar{B} + \bar{C})$

Solution: $\hat{e}_{\bar{A} + \bar{B} + \bar{C}} = \frac{(\bar{A} + \bar{B} + \bar{C})}{|\bar{A} + \bar{B} + \bar{C}|}$

$$(\bar{A} + \bar{B} + \bar{C}) = (3i + 7j - 2k) + (i + 3j + 7k) + (9i - 4j + 6k)$$
$$= 13i + 6j + 11k$$

$$|\bar{A} + \bar{B} + \bar{C}| = \sqrt{13^2 + 6^2 + 11^2} = \sqrt{326} = 18.01$$

$$\therefore \frac{13i}{18.01} + \frac{6j}{18.01} + \frac{11k}{18.01}$$

2. $x = 8t^2$, $y = t^2 - 4t$, $z = t - 1$ where t is time, find the modulus of acceleration at $t = 1$

Solution

The position vector $r = xi + yj + zk$
 $r = (8t^2)i + (t^2 - 4t)j + (t - 1)k$

To find acceleration, we must first find $\frac{dr}{dt}$

$$\frac{dr}{dt} = (16t)i + (2t - 4)j + k$$

$$\frac{d^2r}{dt^2} = 16i + 2j$$

$$\left| \frac{d^2r}{dt^2} \right|_{t=1} = \sqrt{16^2 + 2^2} = 16.12$$

3. $A = 4i + 2j - 4k$
 $B = 8i - 2j + k$
 $C = i + 4j - 3k$

Find the vector triple product $(A \times B) \times C$

Solution

$$(A \times B) = \begin{vmatrix} i & j & k \\ 4 & 2 & -4 \\ 8 & -2 & 1 \end{vmatrix}$$

$$i \begin{vmatrix} 2 & -4 \\ -2 & 1 \end{vmatrix} - j \begin{vmatrix} 4 & -4 \\ 8 & 1 \end{vmatrix} + k \begin{vmatrix} 4 & 2 \\ 8 & -2 \end{vmatrix}$$

$$= -6i - 36j - 24k$$

$$(A \times B) \times C = \begin{vmatrix} i & j & k \\ -6 & -36 & -24 \\ 1 & 4 & -3 \end{vmatrix}$$

$$i \begin{vmatrix} -36 & -24 \\ 4 & 3 \end{vmatrix} - j \begin{vmatrix} -6 & -24 \\ 1 & -3 \end{vmatrix} + k \begin{vmatrix} -6 & -36 \\ 1 & 4 \end{vmatrix}$$

$$= -12i - 42j + 12k$$

$$\therefore (A \times B) \times C = -12i - 42j + 12k$$