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Computer Engineering
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MAT 104
Assignment

Integrate the following with respect to their variables

1. $x^{1/2} \ln x$

$x^{1/2} \ln x \, dx$

$u = \ln x$ $\qquad dv = x^{1/2}$

$\frac{du}{dx} = 1/x$ $\qquad V = \frac{2x^{3/2}}{3}$

$dux = dx$

$du = \frac{dx}{x}$

$$\int u\,dv = uv - \int v\,du$$

$$\int u\,dv = \ln x \times \frac{2x^{3/2}}{3} - \int \frac{2x^{3/2}}{3} \times \frac{dx}{x}$$

$$= \frac{2x^{3/2}\ln x}{3} - \int \frac{2x^{1/2}}{3}\,dx$$

$$= \frac{2x^{3/2}\ln u}{3} - \left(\frac{2x^{3/2}}{3 \times 3/2} + c\right)$$

$$\int x^{1/2} \ln x \, dx = \frac{2x^{3/2}\ln x}{3} - \frac{4x^{3/2}}{9} + c$$

2 $2\cos 6t \cos t$

$\int 2\cos 6t \cos t \, dt$

$\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$A = 6t$

$B = t$

$\cos 6t \cos t = \frac{1}{2}[\cos(6t+t) + \cos(6t-t)]$

$= \frac{1}{2}[\cos 7t + \cos 5t] \, dt$

$\int 2 \times \frac{1}{2}[\cos 7t + \cos 5t] \, dt$

$\int \cos 7t + \cos 5t \, dt$

$\int \cos 7t \, dt + \cos 5t \, dt$

$= \frac{\sin 7t}{7} + \frac{\sin 5t}{5} + C$

$\therefore 2\cos 6t \cos t = \frac{\sin 7t}{7} + \frac{\sin 5t}{5} + C$

3 $\sin^3 x \cos^4 x$

$\int \sin^3 x \cos^4 x \, dx$

$u = \cos x$

$\frac{du}{dx} = \frac{-\sin x}{\cancel{x}}$

$du = -\sin x \, dx$

$dx = \frac{du}{-\sin x}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$= \int \sin x \sin^2 x \, u^4 \times \frac{du}{-\sin x}$

$= \int u^4 \times -\sin^2 x \, du$

$= -\int u^4 \sin^2 x \, du$

$= -\int (1 - \cos^2 x) u^4 \, du$

$= \int (1 - u^2) u^4 \, du$

$= \int (u^6 - u^4) \, dx$

$\int u^6 \, du - \int u^4 \, du$

$$= \frac{u^7}{7} - U^5 + C$$

$$= \frac{\cos x^7}{7} - \frac{\cos x^5}{5} + C$$

$$\therefore \sin^3 x \cos^{\cancel{x}\,4} x = \frac{\cos x^7}{7} - \frac{\cos x^5}{5} + c$$