OLASOJI. OLUWABORI .C

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PETROLEUM ENGINEERING.

1.) Find the limit of the function: $(4x^2 - \sin x) / x^3$ as $x \to 0$

$$\therefore \lim_{x \to 0} \left[\frac{4x^2 - \sin x}{x^3} \right]$$

By direct substitution: $\lim_{x \to 0} \left[\frac{4(0)^2 - \sin(0)}{(0)^3} \right] = \frac{0 - 0}{0}$ undefined

Using L'Hopital's rule:

$$\lim_{x \to 0} \left[\frac{4x^2 - \sin x}{x^3} \right] = \lim_{x \to 0} \left[\frac{8x - \cos x}{3x^2} \right] = \left[\frac{8 + \sin x}{6x} \right]$$

$$\lim_{x \to 0} \left[\frac{8 + \sin x}{6x} \right] = \lim_{x \to 0} \left[\frac{\cos x}{6} \right] = \frac{\cos(0)}{6} = \frac{1}{6}$$

$$\therefore \lim_{x \to 0} \left[\frac{4x^2 - \sin x}{x^3} \right] = \frac{1}{6}$$

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2. Find dy/dx; If $y = \frac{7x^2 \cos 8x}{e^{3x}}$

Let $U = 7x^2$, $V = \cos 8x$, $w = e^{3x}$

$\frac{du}{dx} = 14x$ $\frac{dv}{dx} = -8\sin 8x$ $\frac{dw}{dx} = 3e^{3x}$

$$\frac{dy}{dx} = y \left[\frac{1}{7x^2}(14x) + \frac{1}{\cos 8x}(-8\sin 8x) + \frac{1}{e^{3x}}(3e^{3x}) \right]$$

$$\frac{dy}{dx} = y \left(\frac{2}{x} - 8\cot 8x + 3 \right)$$

$$\frac{dy}{dx} = \left(\frac{7x^2 \cos 8x}{e^{3x}} \right) \left[\frac{2}{x} - 8\cot 8x + 3 \right]$$

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3. If $y = \cos(5x^2 + 6x)$ find dy/dx

$u = 5x^2 + 6x$ $\quad y = \cos u$

$\frac{du}{dx} = 10x + 6$ $\quad \frac{dy}{du} = -\sin u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$

$= 10x + 6 \times -\sin u$

$\frac{dy}{dx} = -10x + 6 \sin u$

since $u = 5x^2 + 6x$

$\frac{dy}{dx} = -10x + 6 \sin(5x^2 + 6x)$

4a. Find the integral of: $\int \frac{3}{4x+1} dx$

$\int \frac{3}{4x+1} dx = 3 \int \frac{dx}{4x+1}$

let $u = 4x + 1$

$\frac{du}{dx} = 4$

$\therefore dx = \frac{du}{4}$

$3 \int \frac{dx}{4x+1} = 3 \int \frac{1}{u} \frac{du}{4}$

$\frac{3}{4} \int \frac{1}{u} du$

$= \frac{3}{4} \ln u + C$

$\therefore \int \frac{3\,dx}{4x+1} = \frac{3}{4} \ln(4x+1) + C$

b.) $\int \frac{dx}{(x^2+49}$

let $x = a\tan\theta$ where $a = 7$

$x = 7\tan\theta$

$\frac{dx}{d\theta} = 7\sec^2\theta$

$dx = 7\sec^2\theta \, d\theta$

$\therefore x^2+49 = (7\tan\theta)^2 + 49$

$= 7\tan^2\theta + 49$

$= 49\tan^2\theta + 49$

$= 49(\tan^2\theta + 1)$

$49 + x^2 = 49(1+\tan^2\theta)$

Recall: $1+\tan^2\theta = \sec^2\theta$

$49 + x^2 = 49\sec^2\theta$

$\int \frac{dx}{(x^2+49)} = \int \frac{7\sec^2\theta}{49\sec^2\theta}\, d\theta$

$= \frac{1}{7}\int d\theta$

$= \frac{1}{7}[\theta] + C$

$\therefore \int \frac{dx}{(x^2+49)} = \frac{1}{7}\tan^{-1}\frac{x}{7} + C$

c.) $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)\, dx$

$= \left[\frac{1}{6}e^{6x} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\cos 8x\right]$

$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{1}{7}\cos 7x + \frac{1}{8}\cos 8x + C$

d.) $\int x\sqrt{9+x^2}\, dx$

let $u = 9 + x^2$

$\frac{du}{dx} = 2x$

$\therefore dx = \frac{du}{2x}$

$\therefore \int x\sqrt{9+x^2}\, dx = \int \cancel{x} \times \sqrt{u} \times \frac{du}{2\cancel{x}}$

$= \frac{1}{2}\int \sqrt{u}\, du$

$= \frac{1}{2}\left[\frac{u^{1/2+1}}{3/2}\right] + C$

$= \frac{1}{2}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right] + C$

$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}u^{3/2}}{3}\right] + C$

$= \frac{1}{3}\left[u^{3/2}\right] + C$

$= \frac{1}{3}(9+x^2)^{3/2} + C$

$\therefore \int x\sqrt{9+x^2}\, dx = \frac{1}{3}\left(\sqrt{9+x^2}\right)^3 + C$