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MAT 104

(1)

find the limit of the function $\left(\frac{4x^2 - \sin x}{x^3}\right)$ $x \to 0$

Using L'Hopital rule, we have

$= \lim_{x \to 0} = \frac{8x - (\cos x)}{3x^2} =$

$\lim_{x \to 0} = \frac{8x - (-\sin x)}{6x} = \lim_{x \to 0} \left(\frac{8 + \sin x}{6x}\right)$

$= \lim_{x \to 0} \frac{(\cos x)}{6(0)} = \frac{\cos(0)}{6(0)} = \frac{1}{6}$

(2)

If $y = (7x^2 \cos 8x) / e^{3x}$. find the derivative of y with Respect to x

let $u = 7x^2$, $v = \cos 8x$, $w = e^{3x}$

$du/dx = 14x$, $dv/dx = -8\sin 8x$ $dw/dx = 3e^{3x}$

$\therefore dy/dx = y\left[\frac{1}{7x^2}(14x) + \frac{1}{\cos 8x}(-8\sin 8x) + \frac{1}{e^{3x}}(3e^{3x})\right]$

$dy/dx = \frac{7x^2 \cos 8x}{e^{3x}}\left[\frac{2}{x} - 8\tan 8x + 3\right]$

(3)

If $y = \cos(5x^2 + 6x)$. find $\frac{dy}{dx}$

let $u = 5x^2 + 6x$, $y = \cos u$

$\frac{du}{dx} = 10x + 6$ $\quad \frac{dy}{dx} = -\sin u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = -\sin u \times (10x + 6)$

$= -\sin(5x^2 + 6x) \times (10x + 6)$

$= -\sin(5x^2 + 6x)(10x + 6)$

(4)

a) $\int \frac{3dx}{(4x+1)}$

$u = 4x + 1$

$\frac{du}{dx} = 4$

$\frac{du}{4} = \frac{\cancel{4}dx}{\cancel{4}}$

$dx = \frac{1}{4} du$

$\int \frac{3}{(4x+1)} dx = \int \frac{3}{u} \cdot \frac{1}{4} du = \int \frac{3}{4u} du = \frac{3}{4} \int \frac{1}{u} du$

$= \frac{3}{4} \ln(u) + C$

$= \frac{3}{4} \ln(4x+1) + C$

b) $\int \frac{dx}{(x^2+49)}$

$= \int \frac{dx}{49\left(\frac{x^2}{49}+1\right)} = \int \frac{dx}{49\left(\frac{x}{7}\right)^2+1} = \frac{1}{49}\int \frac{dx}{\left(\frac{x}{7}\right)^2+1}$

$u = x/7$

$\frac{du}{dx} = 1/7$

$dx = 7du$

$= 1/49 \int \frac{dx}{u^2+1} = 1/49 \int \frac{1}{u^2+1} \cdot 7du = 7/49 \int \frac{1}{u^2+1} du$

Recall, $\int \frac{1}{u^2+1} du$ is a standard integral, so it equates to arctan u

$\frac{1}{7} \times \arctan u + C$

$= \frac{\arctan(x/7)}{7} + C$

ⓒ $\int (e^{6x} + 9x^3 - \sin 7x + \cos 8x)dx$

$\int e^{6x}dx + \int 9x^3 dx - \int \sin 7x\, dx + \int \cos 8x\, dx$

$= \frac{e^{6x}}{6} + \left[\frac{9x^4}{4}\right] - \frac{1}{7}\left(-\cos 7x\right) + \frac{1}{8}\left(\sin 8x\right) + C$

$= \frac{e^{6x}}{6} + \frac{9x^4}{4} + \frac{\cos 7x}{7} + \frac{\sin 8x}{8} + C$

(d) $\int x\sqrt{9+x^2}\,dx$

$\int x\sqrt{(9+x^2)}\,dx$

$u = 9 + x^2$

$du/dx = 2x$

$\frac{du}{2x} = \frac{2x\,dx}{2x}$

$dx = du/2x$

$\int x\sqrt{u}\cdot du/2x$

$= \frac{1}{2}\int\sqrt{u}\,du = \frac{1}{2}\int u^{1/2}\,du$

$= \frac{1}{2}\left[\frac{u^{1/2+1}}{1/2+1}\right] + c$

$= \frac{1}{2}\frac{u^{3/2}}{3/2}$

$\frac{1}{2}u^{3/2} \times \frac{2}{3}$

$= \frac{2}{6}u^{3/2}$

$= \frac{1}{3}u^{3/2} = \frac{1}{3}[9+x^2]^{3/2}$

$= \frac{(9+x^2)^{3\times 1/2}}{3}$

$= \frac{(9+x^2)(\sqrt{9+x^2})^2}{3} + c$